1. The graph of a function $f$ is shown below. Based on the graph, evaluate these limits or explain why they don’t exist.

(a) $\lim_{x \to -\infty} f(x) \quad 1$

(b) $\lim_{x \to -4^-} f(x) \quad -\infty$

(c) $\lim_{x \to -4^+} f(x) \quad 0$

(d) $\lim_{x \to -4} f(x) \quad \text{Does not exist}$

(e) $\lim_{x \to 3^-} f(x) \quad 4$

(f) $\lim_{x \to 3^+} f(x) \quad -1$

(g) $\lim_{x \to 3} f(x) \quad \text{Does not exist}$

(h) $\lim_{x \to 5^-} f(x) \quad 2$

(i) $\lim_{x \to 5^+} f(x) \quad 2$

(j) $\lim_{x \to 5} f(x) \quad 2$

2. At what points is the function $f$ graphed above discontinuous?
3. Evaluate each of the following limits or explain why they don’t exist.

(a) \( \lim_{{x \to 0}} \frac{x^2 + 3x - 4}{x + 2} \) \(-2\)
(b) \( \lim_{{x \to 1}} \frac{x - 1}{x^3 - x^2} \) \(1\)
(c) \( \lim_{{x \to 4}} \frac{x^2 - 2x - 8}{x^2 - 6x + 8} \) \(3\)
(d) \( \lim_{{x \to -3^-}} \frac{1}{x + 3} \) \(-\infty\)
(e) \( \lim_{{x \to -3^+}} \frac{1}{x + 3} \) \(\infty\)
(f) \( \lim_{{x \to \infty}} \frac{1}{x^2} \) \(0\)

4. You’re trying to figure out whether some function \( f \) is continuous at \( x = 0 \). You know that \( f(0) = -1 \) and the right-hand limit is \( \lim_{{x \to 0^+}} f(x) = -3 \). Explain why \( f \) cannot be continuous at \( x = 0 \), regardless of what the left-hand limit is at \( x = 0 \). (Hint: There are two possible cases to consider.)

If \( \lim_{{x \to 0^-}} f(x) \) is also \(-3\), then the overall limit \( \lim_{{x \to 0}} f(x) \) will be \(-3\). This is not equal to \( f(0) \), so \( f \) is not continuous at \( 0 \). Otherwise, the left-hand limit \( \lim_{{x \to 0^-}} f(x) \) is either equal to a number other than \(-3\) or does not exist. Either way, the two one-sided limits at zero do not agree, so \( \lim_{{x \to 0}} f(x) \) does not exist and \( f \) is not continuous at \( 0 \).

5. Find a value for the constant \( a \) such that the function

\[
f(x) = \begin{cases} x^3 + a & \text{if } x \leq 2 \\ x^2 + x & \text{if } x > 2 \end{cases}
\]

is continuous at \( x = 2 \).

In order for \( f \) to be continuous at \( x = 2 \), the two pieces of the function must match up at \( x = 2 \). Setting \( x^3 + a = x^2 + x \) and substituting in \( x = 2 \), we can solve for \( a \) to get \( a = -2 \).