Harris Math Camp 2019, Section 2
Solutions to Practice Problems
Wednesday, September 4

Disclaimer: Generally, these solutions will only contain the final answers to the problems and a brief explanation, not all of the intermediate work that leads there. Ask me (Dylan) or one of the TAs if you don’t understand where the answer comes from!

1. Determine whether each of the following are functions or not on their stated domains.
   
   (a) $f(x)$ is the closest Starbucks to the location $x$, where the domain of $f$ is the surface of the Earth
   
   This is not a function—at some locations $x$, the two closest Starbucks will be exactly the same distance away, so there is more than one value for $f(x)$.

   (b) $f(x)$ is the distance to the closest Starbucks from the location $x$, where the domain of $f$ is the surface of the Earth
   
   This is a function—the issue that arose in part (a) doesn’t happen here, since even if there is more than one closest Starbucks, all of them that are the closest have to be the same distance away.

   (c) $f(x)$ is the value of $y$ satisfying $x^2 + y^2 = 1$ on the domain $[-1, 1]$
   
   This is not a function, since $f(0)$ could be either 1 or −1.

   (d) $f(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ x - 1 & \text{if } x \geq -1 \end{cases}$ on the domain $\mathbb{R}$
   
   This is not a function, since $f(-1) = 1$ according to the first piece of the function and $f(-1) = -2$ according to the second.

   (e) $f(x) = \begin{cases} x^2 & \text{if } x \leq -1 \\ x + 2 & \text{if } x \geq -1 \end{cases}$ on the domain $\mathbb{R}$

   This is a function, as $f(-1) = 1$ regardless of which piece of the function is used.

2. Find the implicit domain and range of each of the following functions.

   (a) $f(x) = x^2 - 1$
   
   Domain: $\mathbb{R}$
   Range: $[-1, \infty)$

   (b) $f(x) = \frac{x^3 - x}{x}$
Domain: $(-\infty, 0) \cup (0, \infty)$ (all real numbers except 0)
Range: $(-1, \infty)$
(Notice that $-1$ is not in the range.)

(c) $f(x) = \sqrt{4 - x^2}$
\begin{tabular}{|l|}
\hline
Domain: $[-2, 2]$ \\
Range: $[0, 2]$ \\
\hline
\end{tabular}

(d) $f(x) = 2^x + 3^x$
\begin{tabular}{|l|}
\hline
Domain: $\mathbb{R}$ \\
Range: $(0, \infty)$ \\
\hline
\end{tabular}

(e) $f(x) = \ln(4x + 2)$
\begin{tabular}{|l|}
\hline
Domain: $(-\frac{1}{4}, \infty)$ \\
Range: $\mathbb{R}$ \\
\hline
\end{tabular}

3. Explain why $a^{\log_a x} = x$ and $\log_a (a^x) = x$ for any positive real numbers $a$ and $x$.

By definition, $\log_a x$ is the power $a$ must be raised to in order to get $x$, which gives us the first equation. For the second equation, $\log_a (a^x)$ is the power $a$ must be raised to in order to get $a^x$, which is clearly $x$. 