1. Find the partial derivatives $f_x$ and $f_y$ of each of the following functions.

(a) $f(x, y) = 3x^3y + 2x^2y^2 + y^4$

(b) $f(x, y) = (x - y)^4$

(c) $f(x, y) = \ln(2xy - x^2)$

(d) $f(x, y) = \frac{x + y}{x - y}$

(e) $f(x, y) = xye^{-x^2}$

(f) $f(x, y) = \frac{x\sqrt{x^2 + y}}{y + 1}$

2. Find the gradient of each function at the specified point. Then plot the function in a 3D grapher (such as GeoGebra) and convince yourself that the direction of the gradient is the direction of greatest increase of the function at that point.

(a) $f(x, y) = 2x^2 + y^2$ at $(1, 1, 3)$

(b) $f(x, y) = e^{-x^2-y^2}$ at $(0, 0, 1)$

(c) $f(x, y) = \frac{x}{x + y}$ at $(1, 0, 1)$

3. Suppose $f$ is a function of two variables whose value only depends on $x$. (In other words, $f(x, y) = g(x)$, where $g$ is a function of one variable.) Use the gradient to explain why the direction of greatest increase of $f$ at any point (when such a direction exists) will always be either the positive $x$ direction or negative $x$ direction.