Teaching Portfolio

Dylan Quintana
Department of Mathematics
University of Chicago

November 14, 2020

Contents

Teaching Philosophy 2
  Statement of Teaching Philosophy ........................................... 2
  Statement on Equity and Inclusion ........................................... 4

Teaching Experience 6
  Teaching Awards .................................................................. 6
  Instructor of Record ................................................................. 6
  College Fellow .................................................................. 7
  Outreach Programs ................................................................. 8
  Other Programs .................................................................... 9
  Teaching Assistant ................................................................. 10
  Mentoring ........................................................................... 10

Sample Course Materials 11
  Syllabus ........................................................................... 11
  Lesson Plan ................................................................-------- 15
  Problem Session Activity ...................................................... 20
  Homework Assignment .......................................................... 24
  Mathematica Assignment ...................................................... 27

Evaluations and Feedback 31
  Student Evaluations ............................................................... 31
  Individual Teaching Consultation ........................................... 34

Pedagogical Development 38
  Qualifications ....................................................................... 38
  Selected List of Teaching Workshops ...................................... 38
Statement of Teaching Philosophy

I approach teaching by considering what my students should take with them when they leave the classroom, then working backwards to figure out the best way to make that happen. My goal as a teacher, broadly speaking, is to prepare my students to solve problems. In the short term, this means having the knowledge necessary to confidently solve specific types of problems falling within the scope of the course. In the long term, this means obtaining an appreciation for general mathematical principles that can be applied to solve all sorts of problems. To achieve my goal, I ask myself what I want my students to remember from each class on several different timescales.

What will they remember one day from now? This is the question most prominent in my mind when planning a lecture. I focus each class session on one or two central ideas, with the objective of getting these ideas lodged in students’ brains instead of filed away in their notebooks. I condense the theme into a statement that’s memorable enough for students to recall it the next day—something concise that still manages to say a lot, such as “The derivative is the instantaneous rate of change.” I reinforce this statement throughout the lecture by demonstrating the idea with pictures of tangent lines, describing applications of the derivative, and discussing with students how they would try to calculate the derivative as a buildup to the formal definition. Along the way, I infuse everything with character: a calculus pun here, an anecdote about the historical controversy over infinitesimals there. All of these aspects combine into one cohesive lesson that students will immediately take away from the lecture, even if it takes more time to master all of the details.

Another method I employ for making each class stand out is actively engaging students in the learning process. I often use strategies such as pairing up students and having them explain their solutions to in-class exercises to each other, providing them an opportunity to internalize concepts they just learned about. In a summer cryptography course for high-school students, my co-instructor and I had groups of students write messages to each other, encrypt them using different ciphers, and work together to decode messages they received from other groups. This allowed students to experience the strengths and weaknesses of each cipher firsthand as they encoded and decoded messages, and they referred back to this activity when these ciphers came up again in later classes. The philosophy of having students drive their own learning is taken to the extreme with inquiry-based learning (IBL). In IBL classes, my role was simply to facilitate discussions between students who were responsible for coming up with their own proofs of theorems and presenting them to each other.

What will they remember one week from now? Seeing math being done in a lecture is one thing, but in order to develop a true understanding, students must work through mathematical problems independently. After a week of grappling with a new concept, students should develop enough familiarity to use it in solving problems. I create homework and exam problems that go beyond testing students’ skills to serve the greater purpose of instilling curiosity by demonstrating the kinds of broad, general questions that they can now answer. (Is the exponential function the only function that’s equal to its own derivative? Let’s find out!)

Other problems are designed for students to build the skill of communicating mathematics. For instance, on a calculus assignment, I tasked students with explaining the principles behind one of the series convergence tests they had learned, then walking through all of the steps of applying the test to determine whether a given series converges. By putting their thought process into words, students have to think about the purpose behind each step they perform instead of pushing numbers and symbols around mechanically. This underscores the importance of effective communication in problem solving; arriving at a solution isn’t very useful without being able to justify the process
that led there. In doing these types of exercises, students are also reflecting on their personal approach to math, enabling them to discover gaps in their knowledge and form new connections.

What will they remember one month from now? To retain knowledge for this long instead of discarding it after an exam comes and goes, students must recognize how it fits into a larger picture. When introducing students to a new concept, I emphasize how it relates to what they’ve learned in the past and how they might use it in the future. I start with an example of a problem that can’t quite be solved using their existing knowledge, let them work out the difficulties preventing them from solving it, then use this example to organically develop the ideas necessary to figure it out. Many a student has remarked that they never really got the point of one method or another until they saw the need for it in context.

Given the interconnected nature of mathematics, students who fall even a little bit behind can quickly find themselves overwhelmed. I combat this by cultivating an atmosphere of collaboration in the classroom. Students working together can cover for each other’s weaknesses, each of them recalling different aspects of a topic they learned about a while ago. They also benefit from having access to a diverse set of approaches to problems and interpretations of ideas. Fostering a sense of community has been especially important in my classes this year, as remote and socially distanced learning makes it more difficult for students to form personal connections. I’ve broken my current calculus class up into small study groups that are (virtually) meeting throughout the quarter to discuss what’s happened in class each week and collaborate on homework assignments. At the end of the course, each group will present a final project on a subject they learned about together. The intent is for every student to find a place for themselves in the overall community of the class, just as they find a place for every concept in the overall structure of mathematics.

What will they remember one year from now? Some of my students will rarely use any specific facts from my class once they’re done with it, but they will all benefit from learning to think like mathematicians, approaching problems by starting with known information and unraveling all of the logical consequences. I promote this type of thinking across a wide range of disciplines by demonstrating an analytical approach to solving problems in the natural sciences, computer science, engineering, economics, and even everyday tasks like scheduling events. I teach my students skills that are applicable beyond mathematics itself, such as using Mathematica to crunch numbers and visualize information. I encourage students to question everything in my class—what technique will work best, why this unexpected answer appears, why they should care about this subtle detail—and formulate answers using their own ideas. Learning how to answer these questions in a rigorous, structured manner is useful to anyone who wants to improve their ability to formulate and pick apart arguments. Whether my students go on to pursue math or not, I provide them with a mode of mathematical thinking that will help them tackle a wide variety of difficulties they’ll face.

What will they remember one decade from now? After this much time has passed, the bulk of the knowledge I imparted to my students will have inevitably faded away. What will remain? I don’t mind if students forget that the derivative of \( x^3 \) is \( 3x^2 \), but I hope they remember what the derivative represents. I don’t mind if students forget the trick used in a graph theory proof, but I hope they remember the variety of phenomena that can be abstracted as graphs. I don’t mind if students forget all of the exercises they struggled through, but I hope they remember the triumph of reaching a solution by their own sheer reasoning. Ultimately, I want to leave my students with a strong impression of what they are capable of doing with everything they learned. When they look back on my class ten years later, I hope my students find that at least in some small way, I’ve changed the way they see the world.
Statement on Equity and Inclusion

I firmly believe that mathematics should be equally accessible to everyone, and that it is my responsibility as a member of the mathematical community to help make this a reality. I work toward this ideal through carefully considered teaching, mentoring, and outreach.

I have served as an instructor and teaching assistant for two summer courses in the Collegiate Scholars Program, which is a program for students in Chicago public high schools that prepares them for college. The program is targeted at students from underfunded schools serving low-income, largely Black and Latino neighborhoods. Statistically, the typical student at one of these schools would not even be expected to graduate from high school, let alone go on to have a successful college career. Working with these students over the course of the summer made me acutely aware of the challenges they face and of how vital programs like the Collegiate Scholars Program are at every level of education.

My courses for the Collegiate Scholars Program were designed to support and engage these students. Most of them had previously encountered math as a collection of facts to be memorized, set in stone long ago and laid out before them devoid of useful context. I want my students to see math as something they have an active role in learning, encouraging them to draw on their own experiences and relate them to the concepts at hand. For instance, during last summer’s class on cryptography, my co-instructor and I had students discuss times when they wanted to communicate a message secretly in order to demonstrate the pervasive importance of encryption. Giving students a personal stake in the subject matter motivated them to continue learning and reinforced the notion that there is a place for them in the study of mathematics.

Inclusivity is a central pillar in the structure of all of my classes, starting from the very first day. I create an inclusive learning environment by informing my students about all of the resources available to aid them on their journey through my class and beyond: office hours, on-campus tutors, disability services. I take care to explicitly describe my expectations and provide transparency for my course policies, which is especially important for first-generation college students and others who are unfamiliar with the unwritten rules of the college classroom. I offer all of my students a standing invitation for one-on-one conversation about anything from their feelings on how the class is being conducted to their long-term career goals. Over the course of several of these conversations with a student taking my calculus sequence, I encouraged him as he progressed from opting out of a higher-level calculus class due to a lack of confidence in his mathematical abilities to declaring a math major by the end of the year.

My teaching and mentoring incorporate strategies that, according to pedagogical research, improve learning outcomes among groups that are traditionally under-represented in math. When providing feedback to individual students, I emphasize my belief that they are personally capable of meeting the standards I have set for the class. This style of feedback has been demonstrated to mitigate the impact of negative stereotypes on the academic performance of groups such as women and racial minorities. During class, I give students opportunities to discuss and refine their ideas in small groups before sharing them with everyone. Providing a low-stakes setting for students to express and refine their thoughts leads to a greater amount of participation from a wider variety of students.

Learning how to implement inclusive teaching practices effectively is an ongoing process. My approach has been shaped by attending workshops exploring inclusivity and implicit bias in the classroom. As a Teaching Fellow for the Chicago Center for Teaching, I exchange thoughts on
these topics with other fellows during group discussions and pass our collective knowledge on to instructors at the university through classes and training programs. This spring, another fellow and I will be running a series of workshops on pedagogy, where matters of diversity, equity, and inclusion are a central focus.

I am aware of the systemic barriers faced by many in the education system, and have endeavored to do my part to help break them down. I plan on continuing what I have started in my future career. I would like to be involved with educational outreach programs for younger students to inspire confidence that they, too, can succeed in math. I would like to apply and further develop teaching methods that make my classes more accessible and fair. And I would like to contribute to a community that values diversity by facilitating training on evidence-based pedagogy for my fellow educators. By acknowledging the background and needs of each individual, I can give everyone the ability to appreciate the beauty and utility of mathematics.
Teaching Experience

Teaching Awards

Graves Prize
I received the Lawrence and Josephine Graves Prize for excellence in undergraduate teaching in Spring 2019. This prize is awarded to graduate students in the University of Chicago mathematics department in their second year as an instructor of record, based on student evaluations and faculty observations.

Physical Sciences Division Teaching Prize
In Spring 2017, I was awarded the Physical Sciences Division Teaching Prize. Every year, this prize is given to three graduate student lecturers or teaching assistants in the University of Chicago Physical Sciences Division for exceptional teaching in undergraduate classes. Nominations are submitted by students, with the winners selected by a faculty committee.

Instructor of Record
As an instructor, I am responsible for conducting three hours of lectures per week, holding office hours, grading exams, assigning final grades, and managing undergraduate graders/junior tutors. I design all course materials, including syllabi, homework assignments, quizzes, exams, and projects.

In the “Calculus” and “Elementary Functions and Calculus” sequences are year-long sequences of classes for first-year undergraduates at the University of Chicago. The former is a more technical sequence intended for students in physical sciences and economics, while the latter is intended for students in social sciences and the humanities.

Calculus 3
Winter 2020, Winter 2019
10 weeks, 30 students

In this course, students gain proficiency in working with sequences and series and are introduced to topics in multivariable calculus, including vectors, partial derivatives, the gradient, optimization, and multiple integration. When I taught this course in Winter 2020, I added a computer component in which students used Mathematica to visualize concepts in multivariable calculus. A weekly problem session was held in a computer lab, during which I delivered a brief lecture on using Mathematica and then gave students supervised time to work on an assignment reinforcing concepts they had learned the previous week. One of these assignments is included in the Sample Course Materials section.

Calculus 2
Autumn 2020, Autumn 2019, Autumn 2018
10 weeks, 30 students

The primary focus of this course is integration, starting with the Fundamental Theorem of Calculus, working through integration techniques, and then discussing applications of integration to area, volume, and center of mass. In addition to lectures, I run a weekly problem session, providing students an opportunity to work on additional practice problems in an environment where they could collaborate with each other and ask me questions. An activity I ran during a problem session is described in the Sample Course Materials section. The current iteration of this course for the Autumn 2020 quarter is partially being conducted
remotely, with problem sessions, office hours, and few of the lectures taking place online and the majority of the lectures happening in a socially distanced classroom. The syllabus for this quarter is available in the Sample Course Materials section.

**Elementary Functions and Calculus 3**  
Spring 2019, Spring 2018  
10 weeks, 15 students

This course covers a hodgepodge of calculus concepts: integration techniques, sequences and series, and multivariable calculus. A good part of the quarter is spent on topics related to differentiation of multivariable functions, skills which many students require for use in future classes.

**Elementary Functions and Calculus 2**  
Winter 2018  
10 weeks, 35 students

This course begins with a prelude on trigonometric functions, then moves to applications of differentiation (graphing and optimization), the Fundamental Theorem of Calculus, and using integration to calculate areas and volumes. Although giving a rigorous proof of every claim is not a goal of this course, most of the key theorems are proven in class, and students are tasked with writing some simple proofs of their own.

**Elementary Functions and Calculus 1**  
Autumn 2017  
10 weeks, 35 students

Starting from no assumed knowledge of calculus, this course develops the notions of limits, continuity, and differentiation. Since students in this class come from a variety of mathematical backgrounds, the first two weeks are dedicated to getting everyone on the same page with pre-calculus concepts such as inequalities and functions.

**College Fellow**

The college fellow program is designed to prepare graduate students in the mathematics department for teaching their own classes. It combines teaching workshops and training with a year of serving as a teaching assistant for a postdoc or senior faculty member.

I was a college fellow for the “Honors Calculus” sequence, offered to advanced University of Chicago first-years who score well on a placement exam. These classes are an introduction to analysis taught in an inquiry-based learning (IBL, also known as the Moore method) format, where students are given scripts with sequences of theorems that they prove themselves and then present to their peers in class. My role as a college fellow was twofold: during class meetings, I facilitated discussion and provided feedback to students; outside of those, I graded homework, held office hours, and conducted a weekly problem session.

**Honors Calculus 3 (IBL)**  
Spring 2017  
10 weeks, 20 students  
Instructor: Chris Henderson
Several advanced calculus concepts are addressed in this course, including sequences, series, multivariable calculus, and sequences of functions. As with all of the Honors Calculus courses, students were responsible for everything written on the board during class; the instructor and I served to provide context for theorems and definitions, point out important examples, and keep students on the right track during their proofs.

**Honors Calculus 2 (IBL)**
Winter 2017
10 weeks, 20 students
Instructor: Chris Henderson

This course begins with the construction of the real numbers, then establishes the usual calculus concepts of limits, continuity, differentiation, and integration in a rigorous fashion. As this is a very fact-paced course, I focused my problem sessions on relating the theoretical concepts students discussed in class to their prior calculus knowledge.

**Honors Calculus 1 (IBL)**
Autumn 2016
10 weeks, 20 students
Instructors: Chris Henderson, Kevin Corlette

Students who place into this course are assumed to be familiar with calculus. This course lays the groundwork for examining calculus on a more theoretical level by exploring the properties of sets, functions, and orders necessary to develop a rigorous notion of the real numbers. Emphasis is placed on learning to communicate mathematics effectively though both writing and speaking.

**Outreach Programs**
I have taught for two courses in the University of Chicago Collegiate Scholars Program, a three-year summer program for underrepresented high-school students from Chicago public schools that prepares them for success in college. I designed both courses with the instructor/co-instructor from the ground up to be interesting introductions to areas of math outside of the typical high school curriculum.

**Introduction to Cryptography (co-instructor)**
Summer 2020
4 weeks, 30 students
Co-instructor: Karl Schaefer

This course was conducted remotely, with two live class sessions each week. Despite the difficulties imposed by working virtually, the class was designed to be as interactive as possible. Between brief lecture segments, much of the class time was dedicated to hands-on activities and discussions, typically involving students encrypting messages using techniques they had learned earlier, then attempting to decipher the messages written by others. My co-instructor and I worked together to design the curriculum, deliver lectures, write and grade homework assignments, and manage in-class activities.
A Problem-Solving Approach to Graph Theory (teaching assistant)
Summer 2017
6 weeks, 20 students
Instructor: Karl Schaefer

This course was meant to broaden students’ awareness of different areas of math by introducing them to graph theory. Students were lectured twice per week on basic concepts in graph theory, then worked in groups to solve problems using those concepts, discovering useful real-word applications. I worked alongside the instructor to plan the curriculum and create homework assignments, and additionally assisted with in-class activities and graded homework.

Other Programs

I have had the opportunity to teach for a variety of other programs that took place at the University of Chicago.

Harris School of Public Policy Math Camp (instructor)
Summer 2019
3 weeks, 150 students

This is an intensive course for incoming Master of Public Policy candidates designed to refresh and reinforce their calculus skills. Students are expected to have prior exposure to calculus, although for many of them this happened at least one job and one degree ago. The course covers calculus starting from limits, focusing on topics important for public policy, namely optimization of functions of one and multiple variables. While the course itself is not graded, students’ ultimate objective is to pass a calculus placement exam at the end of the summer. My duties consisted of designing the curriculum based on input from public policy students and other calculus instructors, giving five two-hour lectures each week, creating daily homework assignments and solutions, and helping teaching assistants plan problem sessions. A lesson plan from one of these lectures is provided in the Sample Course Materials section.

Linear Algebra and Combinatorics (teaching assistant)
Summer 2017
5 weeks, 50 students
Instructor: László Babai

This course is part of the mathematics department’s Research Experience for Undergraduates program, aimed at undergraduates who have just completed their first or second year. It uses a largely problem-driven approach to teach students the basics of linear algebra, combinatorics, and their intersection in spectral graph theory. As a teaching assistant, I conducted two problem sessions each week, which were structured to give students a chance to solve problems and present the solutions to their peers. I also held office hours and graded homework assignments.
Teaching Experience

Teaching Assistant
As an undergraduate at Carnegie Mellon, I was a teaching assistant for two different courses. My responsibilities included leading recitations, holding office hours, grading homework, and assisting with grading exams.

Matrices and Linear Transformations
Spring 2015
15 weeks, 25 students
Instructor: David Handron

This is an introductory linear algebra course mainly concerned with properties of finite, real vector spaces. Topics covered include systems of linear equations, bases, inner products, eigenvalues and eigenvectors, and matrix decompositions.

Concepts of Mathematics
Autumn 2014, Spring 2014, Autumn 2013, Spring 2013
15 weeks, 25 students
Instructors: Tim Flaherty, Mike Picollelli, Gregory Johnson, Mike Picollelli (in reverse chronological order)

This course serves as a primer on formal mathematical proofs for first-year math majors. As such, it focuses squarely on learning proof techniques rather than any particular area of mathematics, though in the process it touches on sets, functions, logic, elementary number theory, combinatorics, and probability.

Mentoring
I have mentored undergraduate students in an individual setting as part of two programs at the University of Chicago: the Directed Reading Program (DRP) and the Research Experience for Undergraduates (REU). Both programs involve students independently learning about a new topic in mathematics, meeting with me weekly to discuss what they’ve learned, then demonstrating their knowledge in a culminating project. Being a mentor involves more than teaching the mathematical content—I have advised students on topics ranging from applying to graduate school to organizing a paper in a logical way. For the DRP, meetings take place over the course of an academic quarter, with students giving a short talk on their reading. The REU takes place during a 5-8 week span over the summer, concluding with writing a research paper. The students I have worked with are listed below along with the title of their talk or paper.

- Valerie Han, *The Graphon as a Limit for Dense Graphs* (REU, Summer 2018)
- Allen Yuan, *Fixed Point Theorems and Applications to Game Theory* (REU, Summer 2017)
Sample Course Materials

Syllabus

This is the syllabus for my current Calculus 2 class. The primary meetings for this class are in person, but office hours and problem sessions are taking place online. I use the syllabus to inform students of the course objectives and policies and preview how class will be conducted. I made sure to address specific challenges students would face this quarter, from social isolation to potentially having to attend large stretches of class meetings virtually due to quarantine procedures.

Calculus II (Math 15200), Section 45
Autumn 2020

In these [insert adjective of your choice] times, the class experience will be somewhat different from normal. But we won’t let a little bit of COVID get in the way of learning! I will try to be lenient with all of you considering the unusual circumstances, and I ask that you do the same for me. All policies in this syllabus are subject to change, including the possibility of moving to a completely online format on short notice. We’ll all get through this weird quarter by supporting each other.

Instructor: Dylan Quintana
Email: dquintana@uchicago.edu
Office Hours: Tu 2–3 PM, Th 4–5 PM, (for homework grading questions)
or by appointment

Course Assistant: Anushka Shivaram
Email: ashivaram1@uchicago.edu

Objectives: By the end of this course, you will be able to:

1. Explain the theoretical basis for integration and relate it to differentiation
2. Understand properties of exponential and logarithmic functions
3. Evaluate integrals using a variety of techniques
4. Apply integration to solve real-world problems

This course is designed to give students an understanding of calculus on a theoretical level and the ability to solve practical problems. Both of these goals are reflected in the learning objectives above.

Lectures: Cummings Life Sciences Center 101, MWF 11:30 AM–12:20 PM
Lectures for the first and last week of class (September 30–October 2, November 30–December 4) will be held over Zoom.

Problem Sessions: Monday 4–5 PM
There will be one problem session each week held over Zoom. We will discuss and work through example problems both as a class and in smaller groups. Attendance is highly encouraged—anything discussed in a problem session could appear on a quiz.

Every problem session starts with an open question period where students can ask any topic covered during the past week of class. I then work through one or two example problems with the entire class before sending them into breakout rooms with a set of additional problems to work on in small groups. The problem session closes with a full-class wrap-up where students discuss the problem-solving strategies they used.
Textbook: *Calculus: One and Several Variables* (Tenth Edition), by Salas, Hille, and Etgen

No homework will be assigned from the textbook aside from optional practice problems, but it will be a useful reference.

Grading: Final grades at the end of the quarter will be based on the following distribution.

<table>
<thead>
<tr>
<th>Component</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>Written homework</td>
<td>10%</td>
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<tr>
<td>Gradarius homework</td>
<td>10%</td>
</tr>
<tr>
<td>Quizzes</td>
<td>50%</td>
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<tr>
<td>Group check-ins</td>
<td>5%</td>
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<tr>
<td>Final project</td>
<td>25%</td>
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Cutoffs for letter grades will be determined at the end of the quarter. No matter how the cutoffs are adjusted, a final grade of 90% will be at least an A-, 80% will be at least a B-, and 70% will be at least a C-. If you have questions on how something was graded, contact me within a week of the graded item being handed back.

Homework: Homework assignments will take two different forms. Written assignments must be uploaded electronically to Canvas; scans or pictures are both fine as long as your work is legible. These will be graded for correctness by the course assistant. Make sure to include all of your work. Each step should be justified in a way that allows the course assistant to easily follow what you are doing.

For computational homework problems, we will be using the online system Gradarius, which can be accessed through the course Canvas page. Step-by-step solutions will be submitted through the Gradarius website and graded automatically. You are allowed to attempt each Gradarius assignment as many times as you like, with only your highest score counting towards your grade.

Gradarius is an online calculus platform that automatically grades homework submissions, including the intermediate work done before arriving at a final answer. I allow unlimited submissions to give students the chance to correct and learn from their mistakes, and they have the ability to contact me directly through Gradarius if they are unsure how to proceed.

Both written and Gradarius homework assignments will be posted on Canvas after each lecture. All six assignments for the entire week (from Monday, Wednesday, and Friday) will be due at the start of class on Wednesday of the following week.

Once per quarter, you may receive an extension of the due date of a week’s worth of homework assignments to one week after their original due date, no questions asked. Send me an email when you decide to use your extension. No late homework will be accepted outside of this extension unless exceptional circumstances (such as testing positive for a certain virus) occur, in which case you should contact me to make arrangements for when to turn in your work.

You are allowed and encouraged to work with other students on the homework, but everything you write down must be your own work. That is, you may collaboratively discuss how to solve the problems, but you may not copy written solutions from anyone else.

Quizzes: There will be seven 15-minute quizzes which will be given at the end of lecture every Friday from October 9 (week 2) to November 20 (week 8). Calculators are not allowed on quizzes. Make-up quizzes will be given at my discretion, as long as advance notice is provided.
The lowest quiz score will be dropped when calculating your final grade.

When I taught this class previously, students reported a great deal of anxiety surrounding midterms, so I decided to replace midterms entirely with weekly quizzes. Feedback so far indicates that quizzes put less pressure on students compared to midterms, encourage them to keep up with the course as it happens rather than trying to cram before an exam, and gives them more opportunities to evaluate their knowledge.

**Study Groups:** To encourage collaboration and provide support throughout the quarter, everyone will be randomly assigned to a study group during the first week of class. Each group is expected to (virtually) meet at least once a week and fill out a short “check-in” survey, to be turned in every Monday. How you use your meeting time beyond completing the weekly survey is up to the group. Consider discussing concepts from class, studying for the upcoming quiz, and working on homework assignments together.

The assigned study groups are intended to fill the void caused by social isolation this quarter. Some students have appreciated having a group readily available for collaboration on assignments, while others haven’t found the groups as helpful and prefer to work alone. Since the amount of mandatory group work is minimal, both types of students can decide how much to rely on the system.

**Final Project:** At the end of the quarter, every study group will work on a final project, which will be presented online during the last week of classes (November 30–December 4). Grades for the final project will have both a group and individual component. More details to come later.

I am using the university-wide mandate of remote final exams as an opportunity to shake up the traditional final format. For the final project, students will give a group presentation about an application of integration, complete with worked examples. After seeing their peers’ presentations, they will individually submit solutions to problems based on these applications.

**Health and Safety:** All students on campus are required to adhere to the guidelines in the UChicago Health Pact in order to promote a safe environment in the classroom.

- Secure face coverings must be worn appropriately at all times at all times while in University buildings.
- Maintain a distance of 6 feet from others.
- Do not attend an in-person class if you feel unwell or are experiencing COVID-19 related symptoms.

Any concerns over inappropriate PPE usage, physical distancing, cleaning/disinfection, or other COVID-19 related public health concerns should be directed to UCAIR. If there is an emergency, call 773-702-8181 or dial 123 on any campus phone. If you were potentially exposed to COVID-19 or your COVID-19 test results come back positive, reach out immediately to C19HealthReport@uchicago.edu.

**Attendance:** Attendance will not be taken and will not factor into your grade, but I strongly recommend attending all of the lectures and problem sessions. It will be difficult to succeed in this course without doing so. If you do miss a day of class, you are responsible for watching the recording or contacting a classmate to determine what you missed.
All in-person class sessions were recorded and made available to students. Quarantining students also had the option to attend lectures live through Zoom, albeit with limited participation.

Students who have been exposed to or who are experiencing symptoms of COVID-19 should contact UChicago Student Wellness immediately to be tested, and reach out to their area Dean of Students to request accommodations for classes until:

• At least 10 days have passed since symptoms first appeared, and
• At least 3 days (72 hours) have passed since recovery—defined as resolution of fever without the use of fever-reducing medications and improvement in respiratory symptoms (e.g., cough, shortness of breath)

Cheating: Don’t.

Any assignment or quiz that shows evidence of cheating, including copying from another source, will automatically receive a zero. Cheating incidents will be reported to the the College’s disciplinary committee.

Electronic Devices: Using laptops, tablets, cell phones, etc. in class is permitted as long as they are not disruptive to other students in any way. However, you should be aware that if I see you staring at a screen (other than the one your Zoom window is on, if applicable) for an entire class instead of paying attention, I will become sad.

Additional Assistance: If you need help understanding any topics in the course, your first resort should be going to office hours or emailing me to arrange another time to meet. The College Core Tutors (core-tutors.uchicago.edu) are another resource available for general conceptual help. It should be noted that these tutors are not affiliated with this specific section of the course or the math department in general, so they may explain things differently than how they were discussed in class.

Accessibility: The University of Chicago is committed to ensuring equitable access to our academic programs and services. Students with disabilities who have been approved for the use of academic accommodations by Student Disability Services (SDS) and need reasonable accommodations to participate fully in this course should follow the procedures established by SDS for using accommodations. Timely notifications are required in order to ensure that your accommodations can be implemented. Please contact me to discuss your access needs in this class after you have completed the SDS procedures for requesting accommodations. SDS can be reached at (773) 702-6000 or disabilities@uchicago.edu.

Recordings: The Recording and Deletion Policies for the current academic year can be found in the Student Manual under Petitions, Audio & Video Recording on Campus.

• Do not record, share, or disseminate any course sessions, videos, transcripts, audio, or chats.
• Do not share links for the course to those not currently enrolled.
• Any Zoom cloud recordings will be automatically deleted 90 days after the completion of the recording.
Lesson Plan

This is an outline for a lesson I taught in my calculus review class for the Harris School of Public Policy in Summer 2019. This was a large summer course of about 150 students with varying mathematical backgrounds designed to bring them up to speed on the calculus concepts they needed to know before starting a master’s program in public policy. Given the limited time frame, it was a very fast-paced course—students met with me for a two-hour lecture and with two teaching assistants for a one-hour problem session five days a week. I coordinated the lectures and the problem sessions via daily meetings with the teaching assistants, and also loosely coordinated the content and pacing with two other instructors teaching different sections. The scope of the course was very ambitious, going from limits to multivariable optimization in less than three weeks. See my website for the complete schedule.

Below is a lesson plan from the sixth lecture of the summer, focused on concavity and local extrema. This lecture was the middle portion of a two-lecture unit on analyzing the graphs of one-variable functions (which started in the second half of the previous day’s class and concluded in the first half of class the following day). I have provided a brief summary of the previous lecture for context, a list of learning objectives for this lecture, and an outline of the lesson itself, complete with commentary describing my motivations behind the plan for each segment. The outline is detailed enough to be followed by another instructor delivering the same lesson.

Background

In class the previous day, we discovered the connection between the derivative of a function and the shape of its graph. Specifically, we showed that the sign of the derivative on an interval determines whether a function is increasing or decreasing on that interval. We worked through a few examples of finding the intervals on which a given function was increasing and decreasing. The class concluded with a discussion of how this information could be used to sketch the graph of the function \( f(x) = \frac{1}{3}x^3 - 3x \). We decided that there were many possible shapes the graph of an increasing function could take, and so the best we could do for \( f(x) = \frac{1}{3}x^3 - 3x \) based on the first derivative alone was to draw a “crappy graph” where we had to guess the shape of the curve beyond making sure it was increasing and decreasing on the appropriate intervals. This set the stage for introducing concavity as a means of describing the shape of a graph in more detail.

Objectives

By the end of this lesson, students will be able to:

- Describe how the shape of a function’s graph is affected by changes in its derivative
- Assess the concavity of a function based on its second derivative
- Explain why the local extrema of a function must occur at its critical points
- Determine the critical points of a function
- Classify critical points using the first and second derivative tests
Outline

1. Summary of the previous class (5 minutes)
   (a) Have a student describe how to determine where a function is increasing and decreasing.
   (b) Recall that we showed yesterday that the function \( f(x) = \frac{1}{3}x^3 - 3x \) is increasing on \(( -\infty, -\sqrt{3})\), decreasing on \((-\sqrt{3}, \sqrt{3})\), and increasing again on \((\sqrt{3}, \infty)\).
   (c) Draw the “crappy graph” from last class where the function is increasing and decreasing on the correct intervals, but with no regard for concavity. Explain that we need to learn more about the function and its derivative to sketch the graph more accurately.

   I begin every class with a summary of the previous class leading into a clear goal to keep in mind for this one. This encourages students to think about how what they’re going to learn will build on what they already know.

2. Concavity (25 minutes)
   (a) Point out that saying a function is increasing or decreasing only uses the sign of \( f' \). If we want more detail about the shape of the graph, we should think about the actual value of \( f' \) and how it changes.
   (b) Have students make a copy of the following chart and give them a minute to fill it in with sketches of what the graph of the function \( f \) looks like in each case:

<table>
<thead>
<tr>
<th>( f'(x) ) is positive</th>
<th>( f'(x) ) is decreasing</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) ) is negative</td>
<td></td>
</tr>
</tbody>
</table>

   Once their four sketches are done, ask students to share their chart with others around them. Then debrief as a class, filling in the chart on the board with input from students. Did each of their graphs have the same general shape as their neighbors’? Hopefully the answer is “yes”—it looks like this information is enough to determine the shape of a graph!

   Having students share their responses with each other is generally useful for giving them more confidence to volunteer in class discussions. In this particular activity, it also drives home the point that these two pieces of information essentially determine what a piece of a graph looks like.

   (c) Introduce the terminology “convex” and “concave” to describe the first and second columns of the chart. Note that these are also known as “concave up” and “concave down”, but we’ll stick to the former terms.

   One of my teaching assistants, who was a public policy student, pointed out that “convex” and “concave” were the terms that would be used in future economics classes these students would take. I stayed consistent with their suggested terminology throughout the course.

   (d) To figure out the shape of a graph given an expression for \( f \), we need a method of figuring out when \( f' \) is increasing or decreasing. How can we do this? Once students
suggest looking at the derivative of \( f' \), define the second derivative and write down the \( f'' \) notation.

Developing the second derivative based on the previous discussion lets students see how it naturally arises in the context of concavity, rather than something we can calculate that just so happens to have a useful interpretation.

(e) Demonstrate how to find the intervals on which \( f(x) = \frac{1}{3}x^3 - 3x \) is convex/concave. Note that this is essentially the same process used to determine where a function is increasing and decreasing, but applied to the second derivative instead of the first. Use the result to make a nicer sketch of the graph of \( f \). (Be sure not to erase the sketch, as it will be referenced later.)

I chose the function \( f(x) = \frac{1}{3}x^3 - 3x \) as an overarching example for two reasons: it is simple enough that students don’t get bogged down in the algebraic details of taking derivatives, yet complex enough to display all of the interesting features I want to highlight (intervals of convexity and concavity, a local minimum and maximum, and an inflection point).

(f) Mention the concept and notation of third derivatives, fourth derivatives, etc. Discuss whether or not the third derivative has a nice geometric interpretation that can be seen at a glance on a graph.

3. Local extrema and inflection points (20 minutes)

(a) Indicate the local minimum and maximum on the graph of \( f(x) = \frac{1}{3}x^3 - 3x \) as other notable features. Define the terms “local minimum” and “local maximum”, emphasizing what “local” means in a mathematical context.

(b) Ask students how they can identify local minima and maxima based on the derivative, using the graph of \( f(x) = \frac{1}{3}x^3 - 3x \) as a motivating example. Determine the coordinates of the local minimum and local maximum of \( f \) to be \((\sqrt{3}, -2\sqrt{3})\) and \((-\sqrt{3}, 2\sqrt{3})\), respectively. Define critical points as places where the derivative of a function is either zero or undefined. Note that the latter type of critical point doesn’t occur in our example, but make a quick sketch of how such a point could be a local extremum.

(c) Discuss: Are all critical points local extrema? Illustrate any counterexamples students come up with.

(d) Define inflection points and describe how to find them. Show that \( f(x) = \frac{1}{3}x^3 - 3x \) has an inflection point at \((0, 0)\). Ask if students have encountered the term “inflection point” outside of the context of a math class, and if so, how they can reconcile the non-technical meaning with our definition.

When introducing terminology (“inflection point” here and “local” above), I have students think about the everyday meaning of words to better remember their mathematical meaning.

4. Break (10 minutes)
In a class this long, taking a break is important so students can recharge their attention spans. I remain available for students to talk to me individually during the break, especially if they have any potentially off-topic questions they want to ask.

5. First and second derivative tests (40 minutes)

(a) Observe that the easiest way to distinguish local minima and local maxima is by looking at a graph of the function. State the first derivative test as a means of capturing this intuition using the derivative.

(b) Task students with finding the critical points of \( f(x) = (x + 1)^2(x - 3)^2 \) and classifying them as local minima, local maxima, or neither using the first derivative test. Once most of them are done, work through the calculations as a class.

By this point in the course, students should be familiar with the classroom convention of being encouraged to collaborate with others on in-class exercises like this one. I give occasional reminders to this effect whenever conversation between students is lacking.

(c) Ask students about the concavity of a function at local minima and maxima to suggest a connection between these two concepts. Formally state this connection as the second derivative test. Provide the function \( f(x) = x^4 \) as a quick example of the second derivative test giving an inconclusive result.

(d) Task students with finding the critical points of \( f(x) = \frac{1}{x^2 - 4} \) and classifying them as local minima, local maxima, or neither using the second derivative test. Instruct them to verify their answer with the first derivative test if they finish early. Once most of them are done, work through the calculations as a class.

Where possible, I give small extensions to in-class problems to students who finish their work quickly to prevent them from having nothing to do as the rest of the class finishes up.

(e) Poll the class about which of the two derivative tests they prefer, now that they’ve worked with them both. Have a few volunteers to explain their responses and use their explanations to make lists of strengths and weaknesses of the first and second derivative tests on the board. As a preview for later in the course, mention that the analogous test we will use for functions of two variables will be similar to the second derivative test.

I generally find classes to be pretty thorough when it comes to collectively putting together lists like these, but once discussion has died down, I will add in any important considerations that I feel the class has overlooked.

6. Graph sketching example and conclusion (20 minutes)

(a) As a summary of everything they’ve learned about analyzing functions so far, have students sketch the graph of \( f(x) = x^4 - 2x^2 \) in as much detail as possible, taking into consideration intervals where the function is increasing/decreasing and convex/concave, local extrema, and inflection points. Work through this example as a class once most of them have finished.
(b) Reiterate that the end goal of learning about all of these concepts is not to sketch accurate graphs (computers can do that!), but rather to be able to link the graphical behavior of a function with information obtained from its first and second derivatives. Have students think of real-life applications of all of the concepts from today and discuss their ideas as a class. Highlight optimization as a particularly useful application that we will focus on during tomorrow’s class.

I always end lessons by having students consciously look back at everything we’ve discussed and placing their knowledge in a broader context. Previewing the next class provides additional motivation for why these ideas are important to the big picture.
Problem Session Activity

For my Calculus 2 class in Autumn 2019, I held a weekly problem session in which students worked on problems that were typically more involved than the ones presented in lectures. During these problem sessions, I split students up into groups of 3–4 and gave each group member a document with exposition about a particular topic and practice problems related to that topic. Students would discuss problem-solving techniques before delving into the problems together. Students generally found these problem sessions to be useful opportunities to deeply engage with concepts in a more relaxed, informal environment than the lectures. This style of problem session returned in my Autumn 2020 Calculus 2 class with a slightly modified virtual format.

This is a document from a problem session early in the quarter on the subject of proofs. I realized that many students came into the class without a firm grasp of what a mathematical proof is and were intimidated by the prospect of having to prove something. I designed this problem session to give students a concrete, accessible idea of what proofs are, working their way up to proving statements related to integration concepts they had recently learned in class. Students were given exactly what appears below with the exception of my comments in boxes, which describe my motivations for including certain elements and details about how I conducted the problem session.

Math 15200, Section 25
Problem Session 2
Tuesday, October 15

Proofs

It’s time to have the talk about the p-word…proofs! Mathematical proofs might sound intimidating, but the idea is quite simple—a proof is an explanation for why something is true. That explanation can include numerical calculations, diagrams, clarifying sentences, and applying other mathematical results you know.

Many statements you may want to prove can be written in the form

“If [assumptions] are true, then [conclusion] is true.”

In order to prove such a statement, you must demonstrate how the conclusion follows from the set of assumptions. This can be tricky, as there’s no formula that will tell you exactly how the proof should be written; you need to figure out how to prove different statements on a case-by-case basis. The good news is that there are some general principles for writing proofs that you will practice now.

A simple example

Let’s start by proving something pretty obvious. Suppose you have a deck of 10 cards, numbered from 1 to 10. You will be proving the following statement:

“The sum of every pair of cards in the deck is less than 20.”

Discuss (or think about) each of the following and write down your responses.
1. First, identify the assumptions (all of the information given to you in the problem) and the conclusion that you’re trying to prove.

2. Suggest a few different ways you could convince someone that the conclusion is true.

3. Of the techniques you listed, which ones would you consider to be a mathematical proof of the statement? Which ones would you consider to be too informal to qualify as mathematical proofs?

The term “proof” as used in math is much stricter than, say, a proof in law or science. A proof is not simply gathering enough evidence for your conclusion to convince you that it’s true; each step in your proof must directly follow from the assumptions and your previous steps, leaving no room for doubt whatsoever. Take this into consideration as you re-evaluate your response to item 3.

Students begin by proving a fairly straightforward statement. This lets them focus on the nature of proofs themselves rather than getting caught up on the mathematical details. As students are discussing their ideas, I go from group to group asking them to describe the proofs they came up with and how mathematical they think each one is. I ask them to weigh in on additional questions such as, “If I draw two cards from the deck 100 times and get a sum less than 20 each time, can I say the statement is true?”

Now repeat steps 1–3 for this statement about the same deck of 10 cards:

“There are two cards in the deck whose sum is 12.”

You may have found that coming up with an airtight proof of this statement is simpler than it was for the first one. This is because the first statement was about every pair of cards in the deck, while this one is about finding one specific pair. When trying to write a proof, it’s helpful to determine whether the result must hold for every possible case or at least one case out of the many options.

In previous conversations with students in my office hours, they were surprised to learn that giving an example can suffice to prove certain statements. This part gets students thinking about the differences in proving existential and universal statements.

A fancier example

Let’s try proving something from calculus. In this section, you will prove the following statement:

“Suppose $f$ is a continuous function on $[a,b]$. Let $m$ be the minimum value of $f$ and $M$ be the maximum value of $f$ on $[a,b]$. Then $m(b-a) \leq \int_a^b f(x) \, dx \leq M(b-a)$.”

First, you will (hopefully) develop some intuition for why the statement is true, which you can then turn into a formal proof. Here’s the step-by-step process.

1. Identify the assumptions and conclusion of the statement to be proven.

2. Sketch a graph of a function $f$ and identify $m$ and $M$ on your graph. What do the quantities $m(b-a)$ and $M(b-a)$ represent? Use your graph to see why the statement is true, at least for the particular function $f$ you decided to draw.

3. Now comes the difficult part. The graph by itself does not constitute a proof of the statement, but you can use what you learned from the graph to explain why the statement should hold
for any function $f$. Figure out which specific properties of $f$, $m$, and $M$ you used to convince yourself that the statement was true in the previous step. Then use these properties, along with other facts and ideas we’ve discussed in class, to write a mathematical proof of the statement.

If you’re still stumped on step 3, remember that $m$ and $M$ are the minimum and maximum values of $f$, and that “minimum” and “maximum” have a specific mathematical meaning. Then try to relate the picture you drew in step 2 to Riemann sums, and think about the connection between Riemann sums and integrals.

This example was carefully chosen for its nice picture proof. The goal is for students to practice a few important proof skills: (1) translating mathematical language into something visual, (2) identifying the properties of the picture they drew that are vital to the proof, and (3) translating their diagram into a written proof. I talk with each group to make sure students realize the difference between proving the statement for the function in their diagram and proving it for all continuous functions $f$.

### A counterexample

Sometimes, rather than proving a statement, you want to show that a particular statement is false. If a statement is making a claim about all of a certain thing (all functions, all integers, all calculus lectures, etc.), you can disprove it by finding a counterexample, which is a specific case that meets the assumptions but doesn’t satisfy the conclusion.

As an example of a counterexample, consider trying to disprove the statement “All positive numbers are odd.” All you have to do is produce a single number, such is 2, which is positive but not odd. Then the promise made by the statement that “if a number is positive, it is odd” clearly does not hold.

Try to come up with a counterexample that disproves the following statement, and show why your counterexample works:

“If $f$ and $g$ are continuous functions on the interval $[a,b]$, then

$$\int_a^b f(x)g(x) \, dx = \left(\int_a^b f(x) \, dx\right) \left(\int_a^b g(x) \, dx\right).$$

Your counterexample should consist of specific functions $f$, $g$ and specific numbers $a$, $b$ for which the conclusion does not hold. Much like in the second deck of cards example, the reasoning should be easier than the proof in the last section since you’re dealing with a specific function rather than properties of functions in general.

### More practice

The best way to become more proficient at writing proofs is by writing proofs. Try applying the three-step process you learned (identifying assumptions, drawing a picture/looking at a specific example, then turning your intuition into a proof) to these problems.

1. Either prove the following variation of the statement from above or disprove it by finding a counterexample. Suppose $f$ be a continuous function on $[a, b]$. Let $m$ be the minimum value
of $f$ and $M$ be the maximum value of $f$ on $[a, b]$. Then

$$m(b - a) < \int_a^b f(x) \, dx < M(b - a)$$

2. For any continuous function $f$, differentiable function $g$, and constant $c$, prove that

$$\frac{d}{dx} \left[ \int_c^{g(x)} f(t) \, dt \right] = f(g(x))g'(x)$$

3. Recall that $U_f^{[a,b]}(n)$ represents the upper sum of $f$ on the interval $[a, b]$ using $n$ rectangles. Prove that $U_f^{[a,b]}(1) \geq U_f^{[a,b]}(2)$ for any continuous function $f$.

4. Prove that $U_f^{[a,b]}(n) \geq U_f^{[a,b]}(2n)$ for any continuous function $f$ and positive integer $n$.

The practice problems steadily increase in difficulty from the first to the last. The first problem is closely related to the statement students proved earlier, so they can take advantage of the intuition they’ve already developed.
Homework Assignment

This is an excerpt from a homework assignment on sequences and series I gave to my Calculus 3 class. Each homework assignment consists of four parts: one part corresponding to each of the week’s lectures (on Monday, Wednesday, and Friday) and a final part with optional ungraded problems. I sourced some problems the textbook for the course (Calculus: One and Several Variables, by Salas, Hille, and Etgen) and came up with the rest myself.

This excerpt contains part of the assignment based on one lecture and all of the extra problems for the week. I have added boxed comments explaining my thought process behind each problem. Aside from these comments, the assignment was presented to students as it appears below.

Part II (Wednesday, January 15)

Textbook problems:

Section 11.3

For problems 13, 21, and 33, state whether the sequence converges, and if it does, find the limit.

13. \( a_n = \left( -\frac{1}{2} \right)^n \)

21. \( a_n = \cos(n\pi) \)

33. \( a_n = \frac{\sin n}{\sqrt{n}} \)

These are computational problems meant to test students’ knowledge of the definition of convergence and ability to find the limit of a convergent sequence.

Non-textbook problems:

1. Use the Monotone Sequence Theorem to show that the sequence defined by \( a_n = \frac{3^{n+1}}{3^n + 1} \) converges. You don’t have to find what it converges to.

2. Use the Monotone Sequence Theorem to show that the sequence defined by \( a_n = \frac{2^n}{n!} \) converges. You don’t have to find what it converges to.

Problems 1 and 2 bridge the gap between knowing the statement of the Monotone Sequence Theorem and being able to apply it in practice. Students must recognize that applying the theorem requires breaking the problem into the two sub-problems of showing that the sequence is monotone and bounded.

In problems 3–7, we will examine the behavior of the geometric sequence \( a_n = r^n \) when the constant \( r \) takes different values.

3. Explain why the sequence \( \{a_n\} \) converges if \( r = 0 \) or \( r = 1 \), and find what it converges to in each case. Explain why \( \{a_n\} \) diverges if \( r = -1 \).

4. Suppose \( r \) is a real number with \( r > 1 \) or \( r < -1 \). Explain why \( \{a_n\} \) diverges for these values of \( r \).
5. Now let \( r \) be a real number with \( 0 < r < 1 \). Use the Monotone Sequence Theorem to show that the sequence \( \{a_n\} \) converges for these values of \( r \).

6. Once again, let \( 0 < r < 1 \). Now that we know \( \{a_n\} \) converges, there is some real number \( L \) such that \( \lim_{n \to \infty} a_n = L \). Define a new sequence \( \{b_n\} \) by \( b_n = ra_n \). Explain why \( \{b_n\} \) is the same thing as \( \{a_n\} \) with the first term removed. This tells us that \( \lim_{n \to \infty} b_n = \lim_{n \to \infty} a_n = L \). Use this equation to show that \( L = 0 \).

7. Finally, take \( -1 < r < 0 \). Use the Squeeze Theorem and the result of the previous problem to show that \( \{a_n\} \) converges to 0 for these values of \( r \).

I like problems 3–7 for testing students on pretty much everything they’ve learned about sequences thus far while proving a useful result. Specific theorems are suggested for problems that require them to prevent students from being overwhelmed by the abstract setting. Problem 6 is meant to be challenging—it uses a trick students haven’t seen in lecture. The problem outlines a solution without revealing it completely.

As a summary, we see that the geometric sequence \( \{r^n\} \)...

- \( \ldots \) converges to 0, if \( -1 < r < 1 \)
- \( \ldots \) converges to 1, if \( r = 1 \)
- \( \ldots \) diverges, if \( r > 1 \) or \( r \leq -1 \)

The sentence introducing the problems and summary at the end are important for ensuring students don’t lose sight of the big picture. I refer back to this result in a future lecture when we discuss geometric series.

Extra Problems

These problems do not count for any points and are not to be turned in. They are here to provide additional practice with explanations and proofs, much like the problem session problems from last quarter. Many of them are trickier than problems you’ll see on homework assignments or exams. Send me an email or talk to me in office hours if you’d like to discuss the solutions.

A student who was also in my Calculus 2 class last quarter told me that she had already taken calculus in high school and wasn’t feeling particularly challenged by the course. As a response, I started including extra problems like these on homework assignments for students who wanted to see interesting theoretical problems that would push them a bit more.

1. Prove that any increasing sequence is bounded below.

2. Let \( \{a_n\} \) be a sequence. Observe that \( \{a_{2n}\} \) is the sequence \( a_2, a_4, a_6, a_8, \ldots \) consisting of every other number in \( \{a_n\} \). If \( \{a_n\} \) converges, does \( \{a_{2n}\} \) necessarily converge? If \( \{a_{2n}\} \) converges, does \( \{a_n\} \) necessarily converge? For each question, either explain why the answer is “yes” or give an example demonstrating that the answer is “no”.

I include conceptual questions similar to problems 1 and 2 on homework and exams. To answer questions of the form “If something has property X, does it necessarily have property Y?”, students must have an understanding of property X and property Y that goes beyond
For the next four problems, consider the sequence with $a_1 = 0$ and $a_{n+1} = \frac{1}{2}(a_n + 2)$ for $n > 1$. This is the sequence you obtain if you start with 0 and keep taking the average of the number you have with 2 to get the next number in the sequence.

3. Show that the sequence $\{a_n\}$ is bounded above by 2. (Hint: If you know that $a_n < 2$ for some value of $n$, you can show that $a_{n+1} < 2$ as well.)

4. Show that the sequence $\{a_n\}$ is increasing. (Hint: You’ll have to use the result of the previous problem.)

5. Prove that the sequence $\{a_n\}$ converges to 2.

6. Prove that $\{a_n\}$ still converges to 2 if $a_1$ is a number other than 0.

Providing a series of results building up to the conclusion of problem 6 makes the problem more digestible. Each of problems 3–6 can be solved using the basic strategies applied elsewhere in the homework assignment, but with an additional twist. (For instance, a savvy student might notice that the limit in problem 5 can be found using the trick from problem 6 from part II of the assignment above.) These problems are a true test of how well students have internalized methods for dealing with sequences.
Mathematica Assignment

One of the greatest challenges in teaching multivariable calculus is helping students visualize graphs of functions in three dimensional space; a two-dimensional chalkboard doesn’t do them justice. For my Calculus 3 class in Winter 2020, I addressed this issue by introducing my students to the basics of Mathematica, providing them with a visual, interactive supplement to the lectures. Every week, I held a problem session in a computer lab, demonstrating some useful features of Mathematica and giving students an assignment incorporating them to explain calculus concepts. Students were encouraged to collaborate to interpret the concepts being demonstrated and debug each other’s Mathematica code.

Assignments were given to students as Mathematica notebooks containing several problems. Students ran code and typed qualitative answers directly into the notebooks, which they turned in to be graded. One assignment is presented here in the form of screenshots with added commentary.

This problem session took place once students had learned about functions of two variables, contour plots, and partial derivatives. The assignment begins with an interactive tutorial on the Mathematica commands needed to plot functions and create contour plots. During the problem session, I gave a live demonstration of how to use these commands, which I summarized here for the sake of students who were unable to attend. Students can evaluate the Mathematica code to produce 3D plots.
The first problem is all about exploring the possibilities of graphs in 3D space. Students were tasked with finding and graphing functions that satisfy certain properties. These properties were chosen to illustrate differences in behavior between single-variable and two-variable functions, such as how the notion of a vertical asymptote could be interpreted in different ways for a two-variable function.
This problem demonstrates the relationship between the slope of a function in a given direction and the spacing of its level curves. Students were first asked to investigate this relationship quantitatively by estimating some slopes, then qualitatively by examining the level curves on the plots displayed above. During the problem session, students found it difficult to parse the calculations required to solve problem 2.2; in retrospect, it would have been helpful to include a diagram clarifying how to estimate the slope of the function.
In the final problem, students answered questions about the partial derivatives of a function using an interactive illustration that I had coded. By adjusting the sliders in the graph, students could see the $x$ and $y$ traces of the graph passing through any given point, which allowed them to determine the signs of the first and second partial derivatives of the function at different points. The interactive nature of the graph was particularly helpful for reasoning about the second partial derivatives, as students could watch the slopes of the traces change as they varied $x$ and $y$. I found that this cleared up a great deal of confusion about the graphical interpretation of the second partial derivatives.
Evaluations and Feedback

Student Evaluations

This is a summary of end-of-quarter evaluations I received from students in courses over the past two years for which I was the instructor of record. Evaluations were conducted by the mathematics department and consisted of both multiple-choice and open-ended responses. In the quantitative portion of the evaluations, students responded to each of the following prompts by giving a rating on a five-point Likert scale, with 1 indicating “strongly disagree” and 5 indicating “strongly agree”.

1. The instructor organized the course clearly.
2. The instructor stimulated my interest in the core ideas of the course.
3. The instructor challenged me to learn.
4. The instructor helped me learn the course content.
5. The instructor was accessible outside of class.
6. The instructor created a welcoming and inclusive learning environment.
7. Overall, the instructor made a significant contribution to my learning.

The summary for each course includes the average of the responses in each of these categories as well as condensed written responses to questions about what I did well in the class and what I could improve on. Complete evaluations are available upon request.

Calculus 3 (Winter 2020, Winter 2019)

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The most frequent types of student responses mentioned the clarity of my explanations (18 students), an engaging and welcoming classroom environment (9 students), and my accessibility in answering questions (7 students). The most common suggestion for improving the course was providing more problems to work through both inside and outside of class (7 students).

Selected student comments:

- “He explained everything clearly and made sure everyone understood before moving on.”
- “He made difficult topics interesting and taught in a way that was easy to understand.”
- “You can tell he really cares about his students.”
- “The problem sessions were really engaging and helpful.”
- “The topics are interesting and make you think about certain concepts in a new way if you were already familiar. He was really helpful in office hours and made class fun.”
- “Dylan was very good at answering everyone’s questions and in a way that made sense.”
- “Dylan made learning fun and explained everything very clearly with lots of examples and multiple modes of explaining if asked. Also appreciated fun lesson titles and his kindness and respect for others. Classroom was a friendly and welcoming environment.”
Calculus 2 (Autumn 2019, Autumn 2018)

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Count/Enrollment</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autumn 2019</td>
<td>24/27</td>
<td>5.00</td>
<td>4.67</td>
<td>4.63</td>
<td>4.91</td>
<td>4.83</td>
<td>4.96</td>
<td>4.96</td>
<td>4.85</td>
</tr>
<tr>
<td>Autumn 2018</td>
<td>32/35</td>
<td>4.93</td>
<td>4.37</td>
<td>4.63</td>
<td>4.80</td>
<td>5.00</td>
<td>4.93</td>
<td>4.70</td>
<td>4.77</td>
</tr>
</tbody>
</table>

The most common responses I received commended my clear exposition (21 students), ability to make class fun and engaging (10 students), and organization of the class (9 students). The primary criticism was that there were not enough examples done in class (9 students), especially of more challenging proof-based problems.

Selected student comments:

- “Concepts were well explained, the class felt safe and open, and even though many different skill levels were represented, there wasn’t any resentment or discomfort in class.”
- “I really enjoyed the problem sessions which took what would have been for me a very boring class to an engaging experience where I actually got to learn about math.”
- “Office hours were very helpful. Very personable and genuinely wants to teach and help.”
- “Having taken [Calculus] BC, it was nice because it wasn’t just a repeat of the course—there were more challenging and interesting things.”
- “He always started at the basics and helped us come to conclusions ourselves.”
- “He was very clear, nice, receptive, and helpful. Overall a great teacher, one of my best math teachers because of how straightforward he was.”
- “VERY open to questions.”
- “Dylan was very organized and good at breaking down the more complex topics into lectures that were really easy to follow.”
- “His ability to have fun while doing math made it easier and more enjoyable to pay attention to.”
- “A fantastic teacher who clearly understood deeply and cared for both the material and his students.”

Elementary Functions and Calculus 3 (Spring 2019)

<table>
<thead>
<tr>
<th>Quarter</th>
<th>Count/Enrollment</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
<th>#4</th>
<th>#5</th>
<th>#6</th>
<th>#7</th>
<th>Average</th>
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</thead>
<tbody>
<tr>
<td>Spring 2019</td>
<td>11/16</td>
<td>4.64</td>
<td>4.45</td>
<td>4.82</td>
<td>4.45</td>
<td>4.73</td>
<td>4.82</td>
<td>4.73</td>
<td>4.66</td>
</tr>
</tbody>
</table>

Many students remarked on the clarity of my explanations (5 students), the quality and quantity of examples done in class (3 students), and my availability and willingness to answer questions (3 students). Some suggested working through more complicated problems in class (3 students) and lowering the difficulty of exams (2 students).

Selected student comments:

- “Dylan is excellent at explaining concepts and cares about his students’ success.”
- “Dylan was an amazing teacher. He was available and made me feel comfortable asking as many questions as I needed during lecture.”
• “Very organized and explained concepts in multiple clear ways.”

• “The material was well structured and I never felt too confused by the way the course progressed.”
Individual Teaching Consultation

During the Autumn 2019 quarter, I requested an Individual Teaching Consultation for my Calculus 2 class from the Chicago Center for Teaching. The consultation consisted of three parts: a pre-observation meeting where I communicated my responsibilities and goals for the course to a teaching consultant, an observation of a lecture by the teaching consultant, and a post-observation meeting where we discussed issues that arose during the lecture and how to address those issues going forward. After the post-observation meeting, the teaching consultant produced the following report summarizing the entire task. I have highlighted sections that illustrate my approach to teaching.
Executive Summary
Dylan Quintana is the instructor of record for MATH 15200: Calculus II. This is a lecture style course comprised of 27 students who meet for 50-minutes three times a week. All the students in the course are first-year undergraduates. It is the second time that Dylan has taught this course. I observed a session of Dylan’s course that took place in Week 6 of Autumn Quarter. The course builds upon the students’ prior knowledge from high school math classes and involves periodic homework assignments and in-class exams. As instructor of the course, Dylan’s responsibilities are to construct a syllabus based upon topics assigned by the department, give lectures, hold office hours, and grade all assignments.

Based on our discussions and my observations in the classroom, Dylan already demonstrates a strong awareness of and ability to use teaching strategies that promote active learning and an inclusive classroom. He periodically involves students through individual problem-solving exercises related to the lecture and frequently requests feedback to ensure adequate pacing and that students are copying the notes from the chalkboard. During his Individual Teaching Consultation, we focused on how Dylan could expand upon these practices to promote active learning and student participation in the classroom even further. We discussed three strategies that he could incorporate into his lecture in order to achieve these aims: cold-calling students, having students do small group and partner activities, and communicating with students about the day’s learning objective(s).

*******************************************************************************
Part I: Pre-Observation Meeting
Date of meeting: 11/4/2019

In our pre-observation meeting, we discussed your goals for the consultation, the structure of the Calculus II course and your role in the course.

You articulated that your goals for this ITC were to: 1) increase overall student participation, 2) understand different levels of student engagement in lectures and 3) increase active learning.

During our meeting, we discussed your current involvement with the CCT and the pedagogical training you’ve engaged in as part of the CCT College Teaching Certificate Program. Because this is the second time that you have taught the course, you were able to identify syllabus adjustments that you made and goals that you achieved based upon math department requirements. We also discussed the background of the students and how the small size of the room could affect student learning and engagement because group activities are limited. Lastly, we discussed the structure for the class session to be observed and its content and concerns regarding time-management.

Part II: Observation and Comments
Date of observation: 11/6/2019

Observation:

At the beginning of class, you gave a brief statement regarding the lecture content for the day and indicated that part of the session would deal with material that was not finished in the previous lecture due to time constraints. You guided the students to show the different items to be covered and indicated the new material for the class session. During this opening lecture, most of the students were engaged in notetaking. You began engaging with new material towards the end of class and ended with a brief summary and a call for homework.

Comments:

A clear roadmap helps students to understand the material and promotes active learning by enabling the student to mentally chart how the session is proceeding and is especially helpful for students across different skill levels. While you already outline the day’s objective with your students orally, writing the roadmap on the board or showing transitions between the material could be helpful. A clearly visible roadmap would also help to address some of your concerns regarding time management and student engagement, as both you and they would be able to glance at it during the class period as a reminder. Your well-paced use of the blackboard also supports student learning by helping students take notes and keep track of the material covered. The end of the class seemed rushed and it might be worthwhile to reevaluate the expectations of coverage versus the time constraints of the session and conclude with a clear summary.

Observation:

Students were asked to do individual exercises three times in the class while you worked on the board. When answers were requested, the same student gave the answer to two of the three
questions. It appeared that during the personal exercises, there was varying participation with the
back section of the room appearing to not participate at all.

Comments:

You already demonstrate a knowledge of and desire to use Classroom Assessment Techniques
(CATs) and there are several ways you can build on this to increase participation in the learning
exercises during the class session. Notifying students that there is a possibility that they could be
notified at random (cold calling) is an option to ensure more participation in the problem activity.
Another method is to divide the students into pairs and have them work on the problems together
with the answer being given to all students at the end of the exercise. An additional option is to
work the problem publicly on the board with different students called at random to help with the
next step. Acknowledging and highlighting individual students’ contributions in these low-stakes
assessments increases student participation and informs students that the expectation is for
students to try with good effort and not have perfect answers.

Observation: While lecturing and using the chalk board, there were times when you spoke very
quickly, making it difficult to follow, and other times when you spoke into the board rather than
facing the class. The students, however, appeared generally engaged in the lecture with the
majority taking notes.

Comments: In a lecture style course, maintaining eye contact and orienting your body towards
the students enhances student learning as it enables them to see and hear all of the material the
instructor is communicating. You showed an awareness of this by stopping to ask for questions
while swapping the boards. Given that this is a math course and the chalkboard is frequently
used, when discussing theoretical concepts, it might be helpful to break between writing work on
the board to face and engage the students.

Part III: Post-Observation Meeting
Date of meeting: 11/11/2019

In our discussion during the post-meeting, we each shared our impressions of how the class
went. Although the students are already actively engaged in many ways, there is still space to
increase student participation and implement more active learning strategies, such as pairing
students into groups to work on problems or cold-calling students. We also discussed the benefits
and concerns regarding cold-calling and worked out a disclaimer to help students understand
how it could be used to help their learning and that they will not be penalized for wrong answers.
We also discussed your concerns regarding time management and how to address this by using
signposts and clear objectives. You indicated that if this session were taught again, you would
have spent less time on the previous material and started the newer material earlier. We also
addressed the issue of student participation and identified the section of the room that appeared
disengaged by neither taking notes nor doing the problem exercises. You plan to explore whether
their lack of engagement correlates to their performance on exams and take-home problem sets.
Overall, you demonstrated an ability to constructively reflect on your teaching and a willingness
to try new learning methods in the classroom to meet your pedagogical goals. This is a strong
indicator that your students will continue to benefit from your efforts to promote active learning
and cultivate an inclusive classroom.
Pedagogical Development

Qualifications

Teaching Fellow, Chicago Center for Teaching
Summer 2020–Spring 2021

Teaching Fellows are an interdisciplinary team of graduate students who work with the Chicago Center of Teaching to promote best teaching practices at the University of Chicago. As a Teaching Fellow, I am mentoring students taking the Course Design and College Teaching class, discussing pedagogical topics with my peers, and working on a project to encourage collaboration between instructors in the mathematics department. In Spring 2021, another Teaching Fellow and I will be facilitating a Fundamentals of Teaching workshop series, which will be open to all instructors at the university and include sessions on constructivist pedagogy, communicating learning objectives, implementing inclusive teaching strategies, and assessing student learning.

College Teaching Certificate
Winter 2021 (expected)

I am on track to earning this certification from the Chicago Center for Teaching by completing the Course Design and College Teaching class, participating in an Individual Teaching Consultation, attending several workshops on pedagogy and inclusive teaching, and writing reflections on how these activities shaped my approach to teaching. The workshops are included in the list below.

Selected List of Teaching Workshops

Creating Inclusive and Accessible Learning Environments
Engaging Students in Remote and Hybrid Teaching
Pedagogical Considerations for Remote and Hybrid Teaching
Summer 2020

This trio of workshops focused on the unique challenges of remote teaching. Attention was given to adapting in-person teaching strategies to an online format, maintaining student engagement without a physical classroom, and creating a welcoming learning environment in which students can thrive during these isolating times.

Exploring Implicit Bias in the Classroom
Winter 2020

This workshop investigated the role of subconscious biases in making classrooms less inclusive for students and explained strategies for mitigating their impact.

Fundamentals of Teaching Mathematical Sciences
Autumn 2019

This was a series of workshops tailored to instructors in the mathematics department. Over the course of four weeks, I discussed the learning processes of math students, inclusivity in math classrooms, creating fair assessments, and specific issues encountered during our teaching careers.
Course Design and College Teaching  
Spring 2019

Course Design and College Teaching is a course offered by the Chicago Center for Teaching to instructors throughout the university. During the quarter, I read literature on pedagogical practices and discussed how to apply those practices to design a course that gives all students an opportunity for learning, culminating in a project in which I designed a syllabus, assessment, and learning activity for a hypothetical new course. Topics included how people learn, creating a welcoming learning environment, keeping students engaged with active learning techniques, and the role and design of assessments.

Workshop on the First Day of Class  
Autumn 2017

This workshop for first-time instructors dealt with writing a syllabus that communicates effective course policies and using the first day of class to set the tone for the rest of the term.

Microteaching Workshop  
Autumn 2016

Microteaching entailed recording a very brief lecture, then watching the video and consulting with a staff member from the Chicago Center for Teaching to discuss how the content of the lecture could be delivered more clearly and effectively.

Teaching@Chicago  
Autumn 2016

This was a one-day conference aimed at first-time graduate student teaching assistants. The conference served as an orientation to the culture of teaching at the University of Chicago through talks and small group discussions, highlighting specific resources available to graduate student instructors at the university.