CLASSICAL TESSELLATIONS AND 3-MANIFOLDS, SPRING 2014, FINAL

DANNY CALEGARI

This final exam was given out in class on Friday, May 30th.

Problem 1. Show that every orientation-preserving hyperbolic isometry g can be written *uniquely* as a product $g = kan$ where (in the upper half-space model)

- k is an elliptic element fixing i;
- a is a hyperbolic element fixing 0 and ∞; and
- n is a parabolic element fixing ∞.

(hint: it might be easier to write $g^{-1} = n^{-1}a^{-1}k^{-1}$)

Problem 2. Show that every closed oriented surface of genus at least 2 has a hyperbolic structure in which it can be tiled by regular right angled pentagons. How many pentagons do you need?

Problem 3. Fix a point p in the hyperbolic plane, and consider a sequence of pairs of points q_i and r_i so that $\mathrm{dist}(p, q_i) \to \infty$ and $\mathrm{dist}(p, r_i) \to \infty$. Show that the angles at p of the triangles $q_i p r_i$ converge to 0 if and only if

$$\mathrm{dist}(p, q_i) + \mathrm{dist}(p, r_i) - \mathrm{dist}(q_i, r_i) \to \infty$$

Problem 4. Suppose Σ is a genus 2 surface with a hyperbolic structure on it. Show that there is an orientation-preserving isometry $i : \Sigma \to \Sigma$ with $i^2 = \mathrm{id}$ (i.e. an *involution*) with 6 fixed points, and identify the quotient orbifold $\Sigma/\langle i \rangle$. (Bonus: show — e.g. by counting dimensions of spaces of hyperbolic structures — that there is a genus 3 surface with a hyperbolic structure that does not admit any non-trivial isometric involution).

Bonus problem 5. What is one thing you liked about this class? What is one thing you didn't like, or wish had been done differently?

Department of Mathematics, University of Chicago, Chicago, Illinois, 60637
E-mail address: dannyc@math.uchicago.edu