## ALGEBRAIC TOPOLOGY, FALL 2017, MIDTERM

## DANNY CALEGARI

This midterm exam was posted online on Friday, October 27, and is due before class Friday, November 3. Collaboration is not allowed, nor is the use of outside materials and textbooks. Hatcher and your class notes may be used to remember definitions, but not to copy calculations or proofs.

Problem 1. Give a CW complex structure and a  $\Delta$ -complex structure on the Klein bottle. Use these structures to compute

- (1) the fundamental group;
- (2) the simplicial homology; and
- (3) the cellular homology

Describe the isomorphism between cellular and simplicial homology.

Problem 2. Show that the complex projective plane  $\mathbb{C}P^2$  is not a nontrivial cover of any other space.

Problem 3. Let X denote a wedge of two circles. What is  $\pi_1(X)$ ? For each of the following covering spaces of X, identify the corresponding subgroup of  $\pi_1(X)$ , and say whether the cover is regular (i.e. normal) or not.



Problem 4. Show that  $H_i(X \times S^n) = H_i(X) \oplus H_{i-n}(X)$  for all i and n (where  $H_i = 0$  for i < 0 by definition).

Problem 5. Give an example of a pair of spaces X, Y with isomorphic fundamental groups, and with homeomorphic universal covers, for which the homology groups of X and Y are different.

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Problem 6. If K is a knot in  $S^3$  (i.e. an embedding of  $S^1$ ) the knot group is the fundamental group of  $S^3 - K$ . Show that the square knot  $K_1$  and the granny knot  $K_2$  indicated by the figures below (taken from Wikipedia) have isomorphic knot groups.



**bonus question:** show that  $S - K_1$  is not homeomorphic to  $S - K_2$ .

Problem 7. State and prove the Five Lemma.

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