Math 161 (31) - Midterm Test 1
Autumn Quarter 2017
Friday, October 20, 2017

Instructions:

- Read each problem carefully.
- Write legibly.
- Show all your work on these sheets. Feel free to use the opposite side.
- This exam has 6 pages, and 5 problems. Please make sure that all pages are included.
- Each problem is worth 10 points.
- You may not use books, notes, calculators, etc. Cite theorems from class or from the texts as appropriate.
- Proofs should be presented clearly (in the style used in lectures) and explained using complete English sentences.

Good luck!
Question 1. Prove using only P1 - P12, indicating where you are using each property, that if $0 < a < 1$ then $a^2 < a$. 
Question 2.  

a) (3 points) State the principle of mathematical induction.

b) (7 points) Prove by induction on \( n \) that

\[ 1 + r + r^2 + \ldots + r^n = \frac{1 - r^{n+1}}{1 - r}. \]
Question 3. Prove that if sets $A$ and $B$ are countable then $A \times B$ is countable.
Question 4.  

a) (3 points) State the definition of a function $f : A \to B$.

b) (7 points) Suppose that $(f \circ g)(x) = x$ for all $x \in \mathbb{R}$. Prove that $g$ is one-one and $f$ is onto.
Question 5.  

a) (2 points) State the ε − δ definition of a limit.

b) (6 points) Define

\[ f(x) = \begin{cases} 
  x, & x \text{ irrational} \\
  -x, & x \text{ rational} 
\end{cases} \]

Prove, using the ε − δ definition of a limit, that

\[ \lim_{x \to 0} f(x) = 0. \]

but that \( \lim_{x \to a} f(x) \) does not exist for \( a \neq 0 \). (Hint: For \( a \neq 0 \), there exists \( \varepsilon > 0 \) such that \( |a| > \varepsilon > 0 \).)