Write clearly, and use a different page for each problem. You are encouraged to work together but problems should be written up individually. No late homework will be accepted. All problem numbers are from Linear Algebra Done Wrong. Numbering will take the form “m.n.r” where m is the chapter, n is the subsection, and r is the problem number. Do the following problems:

Throughout we let \( V \) be a vector space, and \( A : V \to V \) a linear operator.

1. Let \( V_1 \) and \( V_2 \) be subspaces of \( V \).
   
   (a) Prove that \( V = V_1 \oplus V_2 \) if and only if for every \( v \in V \) there exist unique \( v_1 \in V_1 \) and \( v_2 \in V_2 \) so that \( v = v_1 + v_2 \).
   
   (b) Record the definition of linearly independent subspaces from the textbook. Prove that \( V_1 \cap V_2 = \{0\} \) if and only if \( V_1 \) and \( V_2 \) is linearly independent.
   
   (c) Show that if \( V = V_1 \oplus V_2 \), and \( B_k \) is a basis for \( V_k \), \( k = 1, 2 \), then \( B_1 \cup B_2 \) is a linearly independent system in \( V \). (This is an alternative proof of a result from class).

2. Let \( \{\lambda_1, \ldots, \lambda_r\} \) be eigenvalues for the operator \( A \) and let \( E_k = \text{Ker}(A - \lambda_k I) \) be the \( k \)th eigenspace. Prove that \( E_k \cap E_j = \{0\} \) for \( k \neq j \). (Note we have already done the case where \( E_k \) is one-dimensional, but the general proof is similar).

3. Consider the matrix

\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}
\]

(a) Find all real eigenvalues of \( A \). Is the matrix \( A \) diagonalizable? Explain.

(b) Consider \( A \) as a matrix with complex entries. Find all (complex) eigenvalues of \( A \). Is \( A \) diagonalizable?

4. Suppose \( A \) is diagonalizable, with \( A = SDS^{-1} \) and \( \lambda \) is an eigenvalue of \( A \). Prove that \( \dim \text{Ker}(A - \lambda I) = \dim \text{Ker}(D - \lambda I) \).

5. 2.2.3 - 2.2.5

6. 2.2.6

7. 2.2.9

8. 2.2.12

9. 2.2.13