Homework 5 – Due May 3

Write clearly, and use a different page for each problem. You are encouraged to work together but problems should be written up individually. No late homework will be accepted. The numbered problems are from Spivak’s Calculus on Manifolds.

• Let \( f, g : \mathbb{R}^n \to \mathbb{R} \) be differentiable at \( a \in \mathbb{R}^n \). Prove the following identities
  
  (a) \( D(f + g)(a) = Df(a) + Dg(a) \)
  
  (b) \( D(fg)(a) = g(a)Df(a) + f(a)Dg(a) \).

  (c) If \( g(a) \neq 0 \), then
  
  \[
  D\left(\frac{f}{g}\right)(a) = \frac{g(a)Df(a) - f(a)Dg(a)}{[g(a)]^2}.
  \]

  **Remark 1.** For this problem, it might be helpful to look back to chapter 10 in Spivak’s Calculus.

• 2.4

• 2.6

• (a) 2.10 (a), (c), (f). Use only the “basic theorems” from this subsection for this problem.
  
  (b) 2.17 (a), (c), (f)

  (c) 2.28 (a), (d)

• 2.13 (problem 2.12 may be helpful).

• 2.23

• 2.29

• 2.31

• 2.34

• Find and classify the critical points of the following functions

  (a) \( f(x, y) = x^3 + y^3 - 3xy \)

  (b) \( f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 \).