Write clearly, use a different page for each problem and explain all your work. You are encouraged to work together but problems should be written up individually. No late homework will be accepted.

• Consider a function $f: \mathbb{R} \to \mathbb{R}$. Prove that $f$ is continuous at $a \in \mathbb{R}$ if and only if for every sequence $\{a_n\}$ with $\lim_{n \to \infty} a_n = a$, we have $\lim_{n \to \infty} f(a_n) = f(a)$.

• Consider the function $f: (0, 1) \to \mathbb{R}$,

\[
f(x) = \begin{cases} 
0 & x \text{ irrational,} \\
1/q & x = p/q \text{ in lowest terms.}
\end{cases}
\]  \hspace{1cm} (1)

Prove that $f$ is continuous at irrational $x \in (0, 1)$ and discontinuous at rational $x \in (0, 1)$.

Choose six of the following problems:

• 6.1
• 6.3
• 6.4
• 6.5
• 6.7
• 6.12
• 6.14
• 6.17 (part (e) is optional)