Write clearly, and use a different page for each problem. You are encouraged to work together but problems should be written up individually. No late homework will be accepted. The numbered problems are from Spivak’s Calculus on Manifolds.

- Let \( \{V_\alpha\}_{\alpha \in I} \) be a collection of subsets of \( \mathbb{R}^n \) indexed by \( I \). Prove that
  \[
  \left( \bigcup_{\alpha \in I} V_\alpha \right)^c = \bigcap_{\alpha \in I} V_\alpha^c, \quad \left( \bigcap_{\alpha \in I} V_\alpha \right)^c = \bigcup_{\alpha \in I} V_\alpha^c
  \]

- 1.14
- 1.15

- Determine the interior, exterior and boundary of the following subsets on \( \mathbb{R}^n \). Determine whether these subsets are open, closed (or neither) and whether or not they are bounded. Prove your claims.
  
  (a) \( \mathbb{Q} \subseteq \mathbb{R} \)
  
  (b) \( \mathbb{Z} \subseteq \mathbb{R} \)
  
  (c) \( [0, 1) \subseteq \mathbb{R} \)
  
  (d) \( B(0, 1) \setminus \{(0, 0)\} \subseteq \mathbb{R}^2 \).
  
  (e) \( \{(x, \sin \frac{1}{x}) | 0 < x < 1\} \subseteq \mathbb{R}^2 \).

- Prove the following: A set \( S \subseteq \mathbb{R} \) is closed if and only if every Cauchy sequence of elements in \( S \) has a limit that is in \( S \).

- 1.19

- (a) 1.20
  
  (b) Let \( K_1 \) and \( K_2 \) be compact sets. Prove that \( K_1 \cup K_2 \) is compact.

- 1.21