Write clearly, and use a different page for each problem. You are encouraged to work together but problems should be written up individually. No late homework will be accepted. The numbered problems are from Spivak’s Calculus.

- (On real power series). Let \( x_0 \in \mathbb{R} \), and suppose that
  \[
  \sum_{n=0}^{\infty} a_n x_0^n, \quad \sum_{n=0}^{\infty} b_n x_0^n
  \]
  converge. Prove that
  \[
  \sum_{n=0}^{\infty} a_n x^n, \quad \sum_{n=0}^{\infty} b_n x^n
  \]
  converge uniformly on \([-a, a]\) for \(0 < a < |x_0|\) to \(f(x)\) and \(g(x)\), respectively, and that the series
  \[
  \sum_{n=0}^{\infty} c_n x^n, \quad c_n = \sum_{k=0}^{n} a_k b_{n-k},
  \]
  converges uniformly on \([-a, a]\) for \(0 < a < |x_0|\) to the product \(fg\). Hint: use Theorem 23-9.

- Prove that a complex function \( f : \mathbb{C} \to \mathbb{C} \) is continuous if and only if its real and imaginary parts are continuous.

- 25.7
- 25.8
- 25.10
- 25.11
- 26.5
- 26.6
- 26.9