Dynamics and blow-up near solitons for the $L^2$ critical gKdV equation

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Abstract. In this talk we revisit the Cauchy problem for the mass-critical gKdV equation $u_t + (u_{xx} + u^5)_x = 0$ with initial data in $H^1$ close to the soliton. For such initial data, finite-time blow-up solutions were previously obtained in a series of papers by Yvan Martel and myself. In this talk we will show that only three scenarios can occur: either (i) the soliton leaves any small neighborhood of the modulated family of solitary waves in the scaling invariant $L^2$ norm, or (ii) the solution is global and converges to a solitary wave as $t \to +\infty$; or (iii) the solution blows up in finite time in a universal stable regime with blow-up rate $1/t$. This is a joint work with Yvan Martel and Pierre Raphaël.