Landis’ conjecture and related questions

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Abstract. In the late 60’s, E.M. Landis conjectured that if $\Delta u + Vu = 0$ in $\mathbb{R}^n$ with $\|V\|_{L^\infty(\mathbb{R}^n)} \leq 1$ and $\|u\|_{L^\infty(\mathbb{R}^n)} \leq C_0$ satisfying $|u(x)| \leq C \exp(-C|x|^{1+})$, then $u \equiv 0$. Landis’ conjecture was disproved by Meshkov who constructed such $V$ and nontrivial $u$ satisfying $|u(x)| \leq C \exp(-C|x|^{1+})$. He also showed that if $|u(x)| \leq C \exp(-C|x|^{4/3})$, then $u \equiv 0$. A quantitative form of Meshkov’s result was derived by Bourgain and Kenig in their resolution of Anderson localization for the Bernoulli model in higher dimensions. It should be noted that both $V$ and $u$ constructed by Meshkov are complex-valued functions. It remains an open question whether Landis’ conjecture is true for real-valued $V$ and $u$. In this talk I would like to discuss a recent joint work with C. Kenig and L. Silvestre on Landis’ conjecture in two dimensions.