Generic properties of scalar parabolic equations

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Abstract. In this talk, one generalizes the classical Kupka-Smale theorem for ordinary differential equations on $\mathbb{R}^d$ to the case of scalar reaction-diffusion equations. More precisely, one shows that, generically with respect to the non-linearity, the semi-flow of a reaction-diffusion equation on a bounded domain in $\mathbb{R}^n$ or on the torus $\mathbb{T}^n$ has the “Kupka-Smale” property, that is, all the critical elements (i.e. the equilibrium points and periodic orbits) are hyperbolic and all the orbits connecting these critical elements (that is, the intersections of the stable and unstable manifolds of the critical elements) are transverse. The proof uses classical arguments in dynamical systems as well as a property of the singular nodal sets of the solutions. In the particular case of $\mathbb{T}^1$, the semi-flow is generically Morse-Smale, that is, it has the Kupka-Smale property and, moreover, the non-wandering set is finite and is only composed of critical elements. This property is important, since Morse-Smale semi-flows are structurally stable. (Joint work with P. Brunovsky and R. Joly).