Green’s function for second order elliptic equations with singular lower order coefficients and applications

Georgios Sakellaris (Universitat Autonoma de Barcelona)

Abstract. We will discuss Green’s function for second order elliptic operators of the form $Lu = -\text{div}(A\nabla u + bu) + c\nabla u + du$ in domains $\Omega \subseteq \mathbb{R}^n$, for $n \geq 3$. We will assume that $A$ is elliptic and bounded, and also that $d \geq \text{div} b$ or $d \geq \text{div} c$ in the sense of distributions.

In the setting of Lorentz spaces, we will explain why the assumption $b - c \in L^{n,1}(\Omega)$ is optimal in order to obtain a pointwise bound of the form $G(x, y) \leq C|x - y|^{2-n}$. Under the assumption $d \geq \text{div} b$, we will also discuss why this assumption is necessary to even have weak type bounds on Green’s function. Finally, for the case $d \geq \text{div} c$, we will deduce a maximum principle and a Moser type estimate, showing again that the assumption $b - c \in L^{n,1}(\Omega)$ is optimal.

Our estimates will be scale invariant and no regularity on $\partial \Omega$ will be imposed. In addition, $L$ will not be assumed to be coercive, and there will be no smallness assumption on the lower order coefficients.