Math 13100 – Section 58 – Final Exam

March 15, 2016

Information and Directions

- This exam will last 120 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- Partial credit is given for showing your calculations and explaining your thoughts.

Name:

| Problem | Points | Score |
|---------|--------|-------|
| 1 | 8 | |
| 2 | 8 | |
| 3 | 4 | |
| 4 | 16 | |
| 5 | 16 | |
| 6 | 16 | |
| 7 | 16 | |
| 8 | 8 | |
| 9 | 8 | |
| | | |

Problem 1 (8 points)

Consider the inequality

-5 < 2x - 1 < 7

Part 1 (4 points)

Express the set of solutions of this inequality as an interval.

Part 2 (4 points)

For which numbers *a* and *b* does |x - a| < b have the same solutions as the above inequality?

Problem 2 (8 points)

Define the functions

$$f(x) = x^{2} + 1$$
 $g(x) = \frac{\sqrt{x-2}}{x-5}$

Part 1 (4 points)

What is the natural domain of g?

Part 2 (4 points)

What is the value of $(f \circ g)(6)$?

Problem 3 (4 points)

Graph the function $f(x) = (x - 2)^2 - 1$.

For at least three different points on your graph, label the point with its x- and y-coordinates. (*Hint: take the graph of* x^2 *and translate it by the appropriate amounts horizontally and vertically.*)



Problem 4 (16 points)

Part 1 (8 points)

Let f(x) be a function. Give the precise, mathematical meaning of the statement

$$\lim_{x \to 5} f(x) = 2"$$

Part 2 (8 points)

Give a complete proof, without appealing to theorems about continuity, that

$$\lim_{x \to 5} x^2 - 3x - 8 = 2$$

Problem 5 (16 points)

Part 1 (8 points)

Let c be a real number, and let f be a function. Give the precise definition of what it means to say

"f is continuous at c"

(Hint: there are three conditions that must be met.)

Part 2 (8 points)

Are there any discontinuities of the function

$$f(x) = \frac{x^2 - 5x + 6}{x - 3}$$
?

(*That is, can you find any c where f is not continuous at c?*) If so, are they removable discontinuities? Explain your answer or thought process.

Problem 6 (16 points)

In this problem, define the function $f(x) = (x^2 - 7)^5$.

Part 1 (8 points)

Using the chain rule, compute the derivative of f.

Part 2 (8 points)

Find the equation of the tangent line to the graph of y = f(x) at 3, i.e., the tangent line that goes through the point

 $(3, f(3)) = (3, (3^2 - 7)^5) = (3, 2^5) = (3, 32)$

Problem 7 (16 points)

Part 1 (8 points)

Using implicit differentiation, find $\frac{dy}{dx}$ where $x^2y^2 + 4xy = 12y$.

Part 2 (8 points)

Find the equation of the tangent line to the graph of

$$x^2y^2 + 4xy = 12y$$

that goes through the point (2, 1).

Problem 8 (8 points)

Use derivatives to approximate the value of $\sqrt{16.1}$.

(Hint: you are being asked to approximate f(16 + 0.1), where $f(x) = \sqrt{x}$. You should start by finding f(16) and f'(16).)

Problem 9 (8 points)

Each edge of a cube is growing at a rate of 3 cm/sec. How quickly is the volume of the cube increasing when the edges of the cube are exactly 12 cm long?

(Hint: The volume of a cube whose edges have length x is equal to x^3 .)