Math 19620 – Section 50 – Midterm 1

October 22, 2015

Information and Directions

- This exam will last 80 minutes.
- This is a closed-book exam.
- No electronic devices are allowed to be used during this exam.
- Make sure to show all details of your work.

Name: ____

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	

Problem 1 (20 points)

Consider this system of linear equations:

$$\begin{aligned} 1x + 4y + 15z &= 12 \\ 2x - 2y + 20z &= 4 \\ 5y + 5z &= 10 \end{aligned}$$

Part 1 (4 points)

Write down the augmented matrix that represents this system of linear equations.

Part 2 (8 points)

Put the augmented matrix into reduced row echelon form. Circle the pivots.

Part 3 (8 points)

Find all solutions of the system of linear equations.

Problem 2 (20 points)

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$f\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 + 2x_2 \\ 9x_1 - x_2 \\ x_1 + 5x_2 \end{bmatrix}$$

Is this a linear transformation?

If so, find the matrix A such that $f(\vec{x}) = A\vec{x}$. If not, explain how you know it is not linear.

Problem 3 (20 points)

Part 1 (5 points)

Write down the 2×2 matrix D_α that represents counterclockwise rotation by an angle α.

Part 2 (5 points)

Two vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ are perpendicular if and only if their dot product $\vec{v} \cdot \vec{w}$ equals ______.

Part 3 (10 points)

Since D_α is a 2×2 matrix, and $\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ is a 2×1 vector, their product $D_\alpha \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ is a 2×1 vector.

Find a value of α such that the vector $D_\alpha \begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}$ is perpendicular to the vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Problem 4 (20 points)

Consider this 3×3 matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 0 \\ 1 & 0 & 4 \end{bmatrix}$$

Part 1 (10 points)

Does the matrix A have an inverse?

If so, find it. If not, explain how you know there is no inverse.

Part 2 (10 points)

What is the rank of the matrix A?
Show your work, or explain how you know. You can appeal to Part 1 and theorems from the book.

Problem 5 (20 points)

Let the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$, and consider the linear transformation $f : \mathbb{R}^3 \to \mathbb{R}^2$ defined by

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Part 1 (10 points)

Find the kernel of f, also known as the kernel of A.

Part 2 (5 points)

Find the image of f, also known as the image of A.

Your answer should be expressed as the span of some set of vectors.

Part 3 (5 points)

Are the columns of the matrix A linearly independent?

If so, explain how you know. If not, write a non-trivial linear relation between the columns.