**Syllabus: Graduate Algebra I**

This is an Intro to algebra, both commutative and noncommutative. The goal is to cover basic material that any mathematician is supposed to know, regardless of his/her specific field of interest.

- **Home assignments** are due each **Friday**.
  Working on a home assignment jointly with another student is OK provided you indicate the name of that person.
  Working in groups of more than 2 people is not allowed.

- **Grading policy:** Home assignments 30%, Midterm 35%, Final 35%.

- There will be a 20 minute Quiz on Friday, **October 12**, right after class.

- The Midterm (2 hours) is scheduled on **Wednesday, October 24**, 4:30-6:30pm, room Eck. 312.
  Undergraduates who get a ‘C’ or below for the Midterm are strongly recommended to drop the course right away.

- The Final exam (2 hours) will be given during the Exam week, on the date provided by the administration.

- **Office hours:** Monday, 2:30 (right after class), Eck. 402. Please feel free to contact me via email any time: ginzburg@math.uchicago.edu.

- **Grader:** Sundeep Balaji. Email: sundeebp@math.uchicago.edu

- **Prerequisites:**
  1. Definitions of: group, ring, ideal, field, subgroup, group homomorphism, factor group, resp. ring homomorphism, factor ring.
     These notions will be considered as ‘known’, and they will not be defined in class.
  2. Knowledge of linear algebra, especially basic properties of tensor product of vector spaces, and of the standard hermitian inner product on \( \mathbb{C}^n \).

- **Main Topics of the course**
  (not necessarily in the order they will be discussed in class):
  1. The algebra of Quaternions. The double covering map \( SU_2 \to SO_3(\mathbb{R}) \). The groups \( GL_n, SL_n \) and their conjugacy classes. The exponential map \( \exp : M_n(\mathbb{R}) \to GL_n(\mathbb{R}) \).
     Orthogonal and symplectic groups.
  3. The Chinese remainder theorem. Application to Lagrange’s interpolation formula.
  5. Group representations: complete reducibility, orthogonality relations for characters, Structure of the group algebra of a finite group and the Plancherel theorem. Induced representations. Duality and Fourier transform for finite abelian groups.
(6) Classification of irreducible representations of finite groups $\mathbb{S}_n$, $GL_2(\mathbb{F}_q)$, and of compact groups $SU_2$, $SO_3(\mathbb{R})$. Invariant theory. Hilbert’s finite generation theorem.

(7) Applications of the Zorn lemma: equivalence of various definitions of semisimple modules, existence maximal ideals.


(10) Spectrum of an element of an algebra; Spectral theorem. Nullstellensatz for algebras over $\mathbb{C}$. Relation between the notions of spectrum and Jacobson radical of a $\mathbb{C}$-algebra.


• Literature:
  
  – Comprehensive reference text:

  – On linear algebra:

  – On noncommutative algebra:

  – On representation theory of groups and algebras:

  – On geometric and topological aspects of Lie groups (Segal’s contribution in):