Syllabus: Graduate Algebra I

This is an Intro to algebra, both commutative and noncommutative. The goal is to cover basic material that any mathematician is supposed to know, regardless of his/her specific field of interest.

- Home assignments are due each **Friday**.
  Working on a home assignment jointly with another student is OK provided you indicate the name of that person.
  Working in groups of more than 2 people is not allowed.
- Grading policy: Home assignments 30%, Midterm 35%, Final 35%.
- There will be a 20 minute Quiz on Friday, **October 12**, right after class.
- The Midterm (2 hours) is scheduled on
  Wednesday, October 24, 4:30-6:30pm, room Eck. 312.
  Undergraduates who get a 'C' or below for the Midterm are *strongly* recommended to drop the course right away.
- The Final exam (2 hours) will be given during the Exam week, on the date provided by the administration.
- Office hours: Monday, 2:30 (right after class), Eck. 402. Please feel free to contact me via email any time: `ginzburg@math.uchicago.edu`.
- Grader: Sundeep Balaji. Email: `sundeepb@math.uchicago.edu`
- Prerequisites:
  (1) Definitions of: *group, ring, ideal, field, subgroup, group homomorphism, factor group, resp. ring homomorphism, factor ring.*
  These notions will be considered as 'known', and they **will not** be defined in class.
  (2) Knowledge of linear algebra, especialy basic properties of *tensor product* of vector spaces, and of the standard *hermitian inner product* on $\mathbb{C}^n$.
- Main Topics of the course
  (not necessarily in the order they will be discussed in class):
  (1) The algebra of Quaternions. The double covering map $SU_2 \twoheadrightarrow SO_3(\mathbb{R})$. The groups GL_n, SL_n and their conjugacy classes. The exponential map $\exp : \mathbb{M}_n(\mathbb{R}) \to GL_n(\mathbb{R})$. Orthogonal and symplectic groups.
  (2) Modules: simple, semisimple, cyclic, finitely generated, and free modules. Schur's lemma. Finite length modules and the Jordan-Hölder theorem. Left, right, and two-sided ideals of a matrix algebra. Description of modules over a matrix algebra.
  (3) The Chinese remainder theorem. Application to Lagrange's interpolation formula.
  (4) Principal ideal domains (PID). Structure theory of finitely generated modules over PID's. Applications: finitely generated abelian groups, Jordan normal form and Jordan decomposition.
  (5) Group representations: complete reducibility, orthogonality relations for characters, Structure of the group algebra of a finite group and the Plancherel theorem. Induced representations. Duality and Fourier transform for finite abelian groups.

(6) Classification of irreducible representations of finite groups $\mathbb{S}_n$, $GL_2(\mathbb{F}_q)$, and of compact groups SU_2, $SO_3(\mathbb{R})$. Invariant theory. Hilbert's finite generation theorem.
(7) Applications of the Zorn lemma: equivalence of various definitions of semisimple modules, existence maximal ideals.
(8) Jacobson's density theorem. Burnside's theorem. The double centralizer theorem. Schur-Weyl duality. The First Fundamental Theorem of Invariant theory.
(9) The Jacobson radical: equivalence of various definitions. Nakayama lemma. Semisimple algebras. Maschke's theorem.
(10) Spectrum of an element of an algebra; Spectral theorem. Nullstellensatz for algebras over $\mathbb{C}$. Relation between the notions of spectrum and Jacobson radical of a $\mathbb{C}$-algebra.
(11) Wedderburn theory: structure theory of simple and semisimple finite dimensional algebras.

- LITERATURE:

– *Comprehensive reference text:*

S. Lang, *Algebra*. Revised third edition. Graduate Texts in Mathematics, 211. Springer-Verlag, New York, 2002.

– *On linear algebra:*

S. Axler, *Linear Algebra Done Right*, Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1996.

– *On noncommutative algebebra:*

J. Beachy, *Introductory Lectures on Rings and Modules*. London Mathematical Society Student Texts **47**, Cambridge Univ. Press, 1999.

– *On representation theory of groups and algebras:*

P. Etingof, O. Golberg, S. Hensel, T. Liu, A. Schwendner, D. Vaintrob, E. Yudovina, *Introduction to representation theory*. With historical interludes by Slava Gerovitch. Student Mathematical Library, 59. American Mathematical Society, Providence, RI, 2011. viii+228 pp.

– *On geometric and topological aspects of Lie groups* (Segal's contribution in):

R. Carter, G. Segal, and I. Macdonald, *Lectures on Lie groups and Lie algebras*, London Mathematical Society Student Texts **32**. Cambridge University Press, Cambridge, 1995.