MATHEMATICS 2520 ASSIGNMENT - DUE APRIL 2

1. Prove: Let M be an abelian group. M is torsion-free if and only if $Tor_1^{\mathbb{Z}}(M, \mathbb{Q}/\mathbb{Z}) = 0$.

2. Let A be a valuation ring with value group an ordered abelian group G.

a) Describe all the ideals in A.

b)If $G = \mathbb{R}$ with its natural ordering, what is the dimension of A?

c) If $G = \mathbb{R} \oplus \mathbb{R}$, with lexicographic ordering $(s, t) > 0$ if and only if $s > 0$ or $s = 0$ and $t > 0$, what is the dimension of A? (Prove your answers)

3. Show that if A is a DVR with quotient field K, then any localization of A is either A or K.

Atiyah-MacDonald: p.84/6 and p.99/3, 6, 7