4b. Accumulation points and boundary points
Math 20300, Winter 2019

§3.3 (3.3.23 - end of section)

Read the rest of section 3.3, and:

• Learn the definitions of accumulation point, boundary point, and closure. Understand what these definitions do for all the examples we’ve discussed in class.

• The definition of closure is hard to think about. Exercises 3.3.31 and 3.3.32 give ways of computing the closure which are much easier to understand. Learn them.

Practice problems for the end of §3.3:

1. For each of the following subsets of a metric space, find the accumulation points, boundary points, and closure of the set.
   
   (a) $\emptyset \subseteq \mathbb{R}$ (the empty set)
   
   (b) $\mathbb{R} \subseteq \mathbb{R}$
   
   (c) $\mathbb{Q} \subseteq \mathbb{R}$
   
   (d) $\mathbb{Z} \subseteq \mathbb{R}$
   
   (e) $\{0, 1/2, 1/3, 1/4, 1/5, \ldots\} \subseteq \mathbb{R}$
   
   (f) $\{1, 1/2, 1/3, 1/4, 1/5, \ldots\} \subseteq \mathbb{R}$
   
   (g) $[0, 1] \subseteq \mathbb{R}$
   
   (h) $[0, 1) \subseteq \mathbb{R}$
   
   (i) $(0, 1) \subseteq \mathbb{R}$
   
   (j) The set of points $(x, y)$ such that $x^2 + y^2 < 1$, in $\mathbb{R}^2$ with Manhattan metric $d_1$
   
   (k) $\emptyset$ in an arbitrary metric space
   
   (l) $X$ (the whole space) in an arbitrary metric space $X$

2. Be able to prove Theorem 3.3.27.

3. 3.3.31

4. 3.3.32

5. Prove that a set is closed if and only if it equals its own closure.

6. Learn the statements in Exercises 3.3.31 and 3.3.32, and see how they apply to different examples. This should help you understand the notion of “closed set” from a different perspective.