How to find the dimension of a vector space:

Use the previous work to find a basis, then count the # of vectors you have.

How to find a parametrization for a vector space:

Find a basis, like $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$

Then a parametrization would be

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + t\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + u\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}.$$

or $$\begin{cases} x = s + 4t + 7u \\ y = 2s + 5t + 8u \\ z = 3s + 6t + 9u \end{cases}.$$

How to find Cartesian eqns describing the Span of some vectors.

- Put the vectors into the columns of a matrix A.
- Form the augmented matrix corresponding to the linear system $A\begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$
- Row reduce.
- For every row of 0's, add an eqn by setting the expression to the right of the 0's to be 0.

Ex $\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & x \\ 3 & 4 & 1 & y \\ 5 & 6 & 1 & z \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & x \\ 0 & -2 & -2 & y-3x \\ 0 & -4 & -4 & z-3x \end{array}\right]$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & x \\ 0 & 1 & 1 & \frac{y-3x}{-2} \\ 0 & 0 & 0 & z-3x-2y+6x \end{array}\right]$$

So, the eqn is $\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; z+3x-2y=0 \right\}$.

Why this works:

Look at the previous example. Requiring that $c+3a-2b=0$ means that the augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & x \\ 0 & 1 & 1 & \frac{y-3x}{-2} \\ 0 & 0 & 0 & z-3x-2y-6x \end{array}\right]$$

represents a consistent system. Thus

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 1 \\ 5 & 6 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

has a sol'n, that is there exist x, y, z s.t.

$$a\begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + b\begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix},$$

so $\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \text{Span}\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$.