HOMEWORK 7

DUE WEDNESDAY, FEBRUARY 25

FROM JACOB:

Section 6.1 — #2; #6 b

NOT FROM JACOB:

1. Show that if A is a $n \times n$-matrix, then $A \cdot \operatorname{Adj}(A) = \det(A)\,\mathrm{Id}_n$. (Hint: it should not take more than two lines to show why the diagonal entries of $A \cdot \operatorname{Adj}(A)$ are all $\det(A)$. To show all the other entries are zero, try to interpret them as determinants of matrices that you get from A by replacing one of the rows with something else.)

2. a) Find the area of the parallelogram spanned by the vectors

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

b) Using part a), find the area of the parallelogram spanned by the vectors

$$\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

3. Let A be an invertible $n \times n$-matrix. Show that the function $A : \mathbb{R}^n \to \mathbb{R}^n$ is 1-1 and onto.

4. Describe geometrically with pictures the functions associated to the following matrices (like we did in class on Wednesday).

a) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

b) $\begin{pmatrix} \cos(\phi) & -\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$

c) $\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

d) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$