Calderón -Zygmund Analysis Seminar

Monday, November 9, 3:45 pm, Eckhart 308

Astala's conjecture on Hausdorff measure distortion under planar quasiconformal mappings and related removability problems

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In his celebrated paper on area distortion under planar quasiconformal mappings (Acta 1994), Astala proved that if E is a compact set of Hausdorff dimension d and f is K-quasiconformal, then fE has Hausdorff dimension at most $d' = \frac{2Kd}{2+(K-1)d}$, and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure $\mathcal{H}^d(E) = 0$, then $\mathcal{H}^{d'}(fE) = 0$. This conjecture was known to be true if $d' = 0$ (obvious), $d' = 2$ (Ahlfors), and $d' = 1$ (Astala, Clop, Mateu, Orobitg and UT, Duke 2008.) The approach in the last mentioned paper does not generalize to other dimensions.

UT showed that Astala's conjecture is sharp in the class of all Hausdorff gauge functions (IMRN, 2008).

Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions (Acta, 2009?) The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calderón-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt A_p theory.

These results are intimately related to removability problems for various classes of quasiregular maps. I will particularly mention sharp removability results for bounded K-quasiregular maps obtained jointly by Tolsa and UT.