1. (a) (10 points). $0 \leq \dim \text{im} \ C \circ B \circ A \leq 7$.

(b) (10 points). By rank-nullity, $23 \leq \dim \ker C \circ B \circ A \leq 30$.

(c) (10 points). $C \circ B \circ A$ cannot be injective, as $\dim \ker$ cannot be zero. $C \circ B \circ A$ cannot be surjective, as $\dim \text{im}$ cannot be 57. As $C \circ B \circ A$ cannot be injective or surjective, it cannot be invertible.

2. (10 points). Many would work; for instance, $\{1, t, t^2, t^3, t^4\}$.

3. (10 points). $\dim \mathcal{P}_4 = 5$ so $\dim \mathcal{P}_4^* = 5$ as well.

4. (a) (5 points). 4.

(b) (5 points). 1.

(c) (10 points). For example: $\{(t - 3), t(t - 3), t^2(t - 3), t^3(t - 3)\}$. 