Math 312, Midterm 2

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1. (5 points) 51.

2. (5 points) 51 (the dual space has the same dimension as the original vector space; this was also on the first midterm).

3. (10 points) 1; a polynomial with zero derivative is constant.

4. (10 points) 50, by rank-nullity.

5. (Many answers suffice for these problems).
   
   (a) (10 points) No; 0 \not\in A.
   
   (b) (10 points) No; 0 \not\in B.
   
   (c) (10 points) Yes; \langle \cdot, \cdot \rangle is positive-definite.
   
   (d) (10 points) Yes; D is the kernel of the linear map \( f \mapsto \langle f, x^3 + x^2 + 1 \rangle \).
   
   (e) (10 points) No; 0 \not\in E.
   
   (f) (10 points) No; 0 \not\in F.
   
   (g) (10 points) No; 0 \not\in G.

6. (a) (10 points) 49; \( H = \ker ev_0 \cap \ker ev_1 \), where \( ev_x(f) := f(x) \). There is a polynomial \( f \) so that \( ev_0(f) \neq 0 \). By rank-nullity, \( \ker ev_0 \) has dimension 50. As \( ev_1 : \ker ev_0 \to \mathbb{R} \) has nontrivial image (there is a polynomial \( f \) for which \( ev_0(f) = 0 \) and \( ev_1(f) \neq 0 \): for instance, \( x \)), rank nullity again shows that \( \ker ev_1 \) restricted to \( \ker ev_0 \) has dimension 49.

(b) (15 points)

\[
\int_0^1 Df(x)g(x) \, dx = f(x)g(x) \bigg|_0^1 - \int_0^1 f(x)Dg(x) \, dx.
\]

But

\[
f(x)g(x) \bigg|_0^1 = 0
\]
\langle Df(x), g(x) \rangle = \langle f(x), -Dg(x) \rangle.