

## COURSE SYLLABUS: MATH 36501, PERFECTOID SPACES (FALL 2018)

**Basic information.** The course meets 3:30-4:20 MWF, in Ryerson 358, during the fall 2018 quarter.

Please note that the first meeting of the course will be Monday, Oct. 8; that is, there will be no meetings the first week of class.

**Instructor.** Akhil Mathew, amathew@math.uchicago.edu.

**Overview.** This will be an introductory topics course aiming to cover some aspects of the theory of *perfectoid spaces* [6, 5]. The course will begin with some foundational aspects of the theory, such as:

- (1) Nonarchimedean geometry, in the formulation of Huber's adic spaces [2, 1] (at least in the case of Tate or Banach rings).
- (2) Almost mathematics and the almost purity theorem.
- (3) Perfectoid algebras and the tilting equivalence.
- (4) Étale cohomology of rigid analytic and adic spaces [4].

Since their introduction perfectoid spaces have led to a wide array of applications; the tentative plan for the remainder of the course is to discuss the following:

- (1) The perfectoid approach to  $p$ -adic Hodge theory and the Hodge-Tate decomposition, after [7].
- (2) The applications to moduli of  $p$ -divisible groups, after [8].

**Prerequisites.** The primary prerequisite for the course is familiarity with the language of schemes. Familiarity with the language of sites, sheaves, and étale cohomology will be useful in parts of the course.

**Office hours.** I will try to hold office hours once a week (my office is Eckhart 326). The timing for office hours will be decided during the course.

**Resources.** In addition to the cited literature, the following resources may be especially useful to attendees of this course:

- (1) The lecture notes and videos from the 2017 Arizona Winter School, <http://swc.math.arizona.edu/aws/2017/index.html>.
- (2) Bhargav Bhatt's lecture notes (from a course at the University of Michigan in fall 2017). For the notes see <http://www-personal.umich.edu/~bhattb/teaching/mat679w17/>
- (3) Notes from the Stanford seminar on perfectoid spaces, <http://math.stanford.edu/~conrad/Perfseminar/>
- (4) Torsten Wedhorn's notes on adic spaces, <https://www2.math.uni-paderborn.de/fileadmin/Mathematik/People/wedhorn/Lehre/AdicSpaces.pdf>

## REFERENCES

1. R. Huber, *Continuous valuations*, Math. Z. **212** (1993), no. 3, 455–477. MR 1207303
2. ———, *A generalization of formal schemes and rigid analytic varieties*, Math. Z. **217** (1994), no. 4, 513–551. MR 1306024
3. Roland Huber, *Étale cohomology of rigid analytic varieties and adic spaces*, Aspects of Mathematics, E30, Friedr. Vieweg & Sohn, Braunschweig, 1996. MR 1734903
4. ———, *Étale cohomology of rigid analytic varieties and adic spaces*, Aspects of Mathematics, E30, Friedr. Vieweg & Sohn, Braunschweig, 1996. MR 1734903
5. Kiran S. Kedlaya and Ruochuan Liu, *Relative  $p$ -adic Hodge theory: foundations*, Astérisque (2015), no. 371, 239. MR 3379653
6. Peter Scholze, *Perfectoid spaces*, Publ. Math. Inst. Hautes Études Sci. **116** (2012), 245–313. MR 3090258
7. ———,  *$p$ -adic Hodge theory for rigid-analytic varieties*, Forum Math. Pi **1** (2013), e1, 77. MR 3090230
8. Peter Scholze and Jared Weinstein, *Moduli of  $p$ -divisible groups*, Camb. J. Math. **1** (2013), no. 2, 145–237. MR 3272049