Rational Points of Generic Curves in Positive Characteristics

It follows from results in Teichmüller Theory that generic curves of type \((g, n)\) in characteristic zero have only \(n\) rational points that come from the tautological points. Using the theory of weighted completion, we prove the analogous result in positive characteristics. The theory of weighted completion was developed by Richard Hain and Makoto Matsumoto. It is a variant of relative completion due to Deligne and can be used to “linearize” a profinite group such as the arithmetic mapping class groups.

Hain used it to show that the section conjecture holds for the generic curve of type \((g, 0)\) in characteristic zero for \(g > 2\). Using comparison theorems, we can also prove the analogous result in positive characteristics.