The University of Chicago
ALGEBRAIC GEOMETRY SEMINAR

Tuesday, November 22nd, 2016
4:30 – 6:00 pm, Eckhart 312
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Unramified correspondences and torsion of elliptic curves

I will report on the results of an ongoing project which we began some years ago with Yuri Tschinkel and continued with Hang Fu and Jin Qian.

We say that a smooth projective curve C dominates C' if there is nonramified covering $\tilde{C}$ of C which has a surjection onto C'. Thanks to Bely's theorem we can show that any curve C' defined over $\overline{\mathbb{Q}}$ is dominated by one of the curves $C_n, y^n - 1 = x^2$. Over $\overline{\mathbb{F}}_p$ any curve in fact is dominated by C_6 which is in a way also a minimal possible curve with such a property. Conjecturally the same holds over $\overline{\mathbb{Q}}$ but at the moment we can prove only partial results in this direction.

There are not many methods to establish dominance for a particular pair of curves and the one we use is based on the study of torsion points and finite unramified covers of elliptic curves.

In fact for any ellitpic curve E over the complex numbers there is a uniquely defined subset of 4 points in $\mathbb{P}^1$ modulo projective transformations defining the curve.These points correspond naturally to a subgroup of points of order 2 in E and there is a well defined (modulo projective transformation) subset of the images of torsion points from E in $\mathbb{P}^1$. The corresponding subsets PE in P^1 for different elliptic curves E, E' have finite intersection and in many cases we can show that such intersection consists of one point. On the other hand there are such subsets with intersection at least 22. It raises a question about the existence of a universal upper bound for such intersections. This question is somewhat related to a question of Serre in the theory of Galois representations.

Any subset PE defines a bigger subset SE in $\mathbb{P}^1$ by closing it by elliptic division. Namely SE contains PE and PE' for any E' defined by four points in SE.

In the case when E is defined over $\bar{Q}$ we show that SE is projectively equivalent to $\mathbb{P}^1(KE)$ where KE is an infinite extension of Q which is not equal to $\overline{\mathbb{Q}}$ and varies for different elliptic curves over $\overline{\mathbb{Q}}$.