Problem 1

Definition: Let $a, b \in \mathbb{Z}$. We write $a \mid b$ (and say “$a$ divides $b$”) if there exists a $k \in \mathbb{Z}$ such that $a \cdot k = b$.

Which of the following are true? (Use the definition above!)

(a) $4 \mid 12$ 
(b) $4 \mid 13$ 
(c) $4 \mid (-12)$ 
(d) $(-4) \mid 12$ 
(e) $(-4) \mid (-12)$ 
(f) $1234 \mid 1$ 
(g) $1 \mid 1234$ 
(h) $1 \mid 0$ 
(i) $0 \mid 1$ 
(j) $0 \mid 0$

Problem 2

What are all the divisors of 24? What are all the divisors of 37? What are all the divisors of 0?

Problem 3

Let $a, b, c \in \mathbb{Z}$. Which of the following are true?

(a) If $a \mid b$, then $a \mid bc$. 
(b) If $a \mid bc$, then $a \mid b$. 
(c) If $a \mid b$ and $a \mid c$, then $a \mid (b + c)$. 
(d) If $a \mid b$ and $a \mid (b + c)$, then $a \mid c$. 
(e) If $a \mid b$ and $b \mid c$, then $a \mid c$. 
(f) If $a \mid b$ and $a \mid c$, then $b \mid c$. 
(g) If $a^2 \mid b^2$, then $a \mid b$. 
(h) If $a \mid b$, then $a^2 \mid b^2$. 