Problem 1
Consider the following question: “Let $\star$ be the operation defined by $a \star b = (2ab - a - b)^2$. Find $1 \star (2 \star 3)$.”

(a) What is the answer?

(b) Is $\star$ commutative? Is it associative?

(c) Is $\star$ a function? If so, what are its domain and range?

Problem 2
Let’s work with “one’s digit arithmetic.” Consider the set $\mathbb{Z}_{10} = \{0, 1, \ldots, 9\}$. We can add and multiply, e.g.,

$$1 + 2 = 3, \quad 6 + 9 = 5, \quad 8 + 2 = 0, \quad 2 \cdot 3 = 6, \quad 5 \cdot 7 = 5, \quad 2 \cdot 0 = 0$$

Which of the properties A1–A4, M1–M4, D hold?

Problem 3
Which elements of $\mathbb{Z}_{10}$ have additive inverses? Which elements of $\mathbb{Z}_{10}$ have multiplicative inverses?

Problem 4
Can you make sense of any of these in $\mathbb{Z}_{10}$?

(a) $5 - 2, \quad 3 - 5, \quad 0 - 4, \quad 1/2, \quad 1/3, \quad 1/4, \quad 2/2, \quad 3/2, \quad 4/2$

(b) $\sqrt{1}, \quad \sqrt{2}, \quad \sqrt{3}, \quad \sqrt{4}, \quad \sqrt{5}, \quad \sqrt{6}, \quad \sqrt{7}, \quad \sqrt{8}, \quad \sqrt{9}$

(c) $\log_3 7, \quad \log_2 2, \quad \log_2 5$

(d) Can you solve $x^2 + x + 4 = 0$ in $\mathbb{Z}_{10}$? Does the quadratic formula work?