1. Divisibility
   (a) Show that if \( b \in \mathbb{Z} \), then \( 1|b \) and \( (-1)|b \).
   (b) Show that if \( a|b \) and \( a|c \) and if \( x \) and \( y \) are any two integers, then \( a|(b \cdot x + c \cdot y) \).

2. \( \mathbb{Z}_{11} \) table
   (a) Fill in the arithmetic tables for \( \mathbb{Z}_{11} \).
   (b) What are the additive inverses for each element that has one?
   (c) What are the multiplicative inverses for each element that has one?
   (d) In \( \mathbb{Z}_{11} \), solve the equation \( 5x + 4 = 2 \).
   (e) In \( \mathbb{Z}_{11} \), solve the equation \( 6x + 5 = 1 \).
   (f) In \( \mathbb{Z}_{11} \), explain why the equation \( x^2 = 2 \) has no solution.

3. Congruence arithmetic
   (a) Describe all of the integers \( x \) such that \( x \equiv 3 \pmod{8} \). List at least 5 examples of such integers \( x \) (including at least two negative integers), and give a formula that describes all such \( x \).
   (b) Describe all of the integers \( x \) such that \( x \equiv 2 \pmod{11} \). List at least 5 examples of such integers \( x \) (including at least two negative integers), and give a formula that describes all such \( x \).
   (c) Use congruence arithmetic to solve \( 2x + 6 \equiv 5 \pmod{7} \).
   (d) Use congruence arithmetic to solve \( 2x + 7 \equiv 5 \pmod{8} \).
   (e) Use congruence arithmetic to solve \( 2x + 8 \equiv 5 \pmod{9} \).
   (f) Make up and solve your own congruence problems.
   (g) Prove that if \( a \equiv b \pmod{n} \) and \( c \equiv d \pmod{n} \), then \( a \cdot c \equiv b \cdot d \pmod{n} \).

4. Base 8
   Suppose we wrote integers in base 8 (instead of base 10). The digits in base 8 are \( \{0, 1, 2, 3, 4, 5, 6, 7\} \). If \( x \in \mathbb{Z} \), we find digits \( a_n, \ldots, a_1, a_0 \in \{0, 1, 2, 3, 4, 5, 6, 7\} \) such that:
   \[
   x = a_n \cdot 8^n + \cdots + a_2 \cdot 8^2 + a_1 \cdot 8 + a_0
   \]
   (a) If the following integers are given in base 10, write them in base 8:
      i. 27
      ii. 66
      iii. 999
   (b) How many integers have 2-digit representations in base 8?
   (c) Find (and prove) a rule for divisibility by 7 in base 8.
   (d) Find (and prove) a rule for divisibility by 9 in base 8.
   (e) Play the Palindrome Game in base 8.