1. The House Problem
   On a street, the houses are numbered consecutively from 1 to $n$. A homeowner notices that the sum of the house numbers down the street is equal to the sum of the house numbers up the street. What is the homeowner’s house number?

2. Pell’s Equation
   For a fixed positive integer $n$, consider the equation $x^2 - ny^2 = 1$.
   (a) Show that if $n$ is a perfect square, then the only integer solution is the trivial solution $(1, 0)$.
   (b) Find all integer solutions when $n = 2$.
   (c) For non-square values of $n$, show that there is a solution, and find the smallest values of $x$ and $y$ that are solutions.

3. Diophantine Equations
   (a) Find all integer solutions to the equation $x^2 = y^2 + 17$.
   (b) Find all integer solutions to the equation $x^2 = y^2 + 24$.
   (c) Find all integer solutions to the equation $x^2 = y^2 + n$ for a given integer $n$.
   (d) Find all integer solutions to the equation $x^3 + y^3 = z^2$. (Hint: Start with particular values of $z$.)
   (e) Find three numbers such that their sum is a square and the sum of any two of them is also a square.

4. More Pythagoras
   Show that all \textit{rational} triples $(a, b, c)$ that satisfy $a^2 + b^2 = c^2$ are of the form $a = (t^2 - 1)r$, $b = (2t)r$, and $c = (t^2 + 1)r$, where $r$ and $t$ are rational numbers.
5. Iterating functions

For function defined as $f : \mathbb{N} \to \mathbb{N}$, consider what happens to a given integer if we iterate the function. That is, given an $n \in \mathbb{N}$, what is $f(n)$? And $f(f(n))$? And $f(f(f(n)))$? Etc. And what happens for different starting values of $n$?

For each of the functions below, consider the following questions:

(a) What happens to each integer $n$? (Yes, this is vaguely phrased. Explore.)

(b) Does the algorithm ever terminate in a fixed point or a loop? That is, are there values $x \in \mathbb{N}$ such that $f(x) = x$, creating a fixed point (and thus terminating the algorithm if we ever reach $x$)? Or, are there values $x_1, \ldots, x_k$ such that $f(x_i) = x_{i+1}$ for $i = 1, \ldots, k - 1$, but $f(x_k) = x_1$, thus creating a loop (and in some sense also terminating the algorithm)?

(c) Given a starting value $n \in \mathbb{N}$, how many step of iteration are required to reach a termination point?

(d) Given a fixed length of the iteration (that is, a number of steps to reach termination), how many values of $n \in \mathbb{N}$ are there that take that many steps?

(e) What patterns and conjectures do you have about your observations from the previous questions?

Okay, here are the functions:

- $f(n) = \begin{cases} n - 1, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

- $g(n) = \begin{cases} 2n, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

- $h(n) = \begin{cases} n - 1, & \text{if } n \text{ is throdd} \\ \frac{n}{3}, & \text{if } n \text{ is threeven} \end{cases}$

- $k(n) = \begin{cases} \frac{n}{p}, & \text{if } n \geq 2 \text{ and } p \text{ is the largest prime such that } p \mid n \\ 1, & \text{if } n = 1 \end{cases}$

- $C(n) = \begin{cases} 3n + 1, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$

- Make up your own function!