1. Investigate powers in $\mathbb{Z}/m\mathbb{Z}$. Pick some small $m$ and compute powers of elements. Make a guess about when you can get all nonzero elements as powers of a single class. Can you make any headway in proving your guess?

2. Let $p$ be a prime. Show that the only class(es) in $\mathbb{Z}/p\mathbb{Z}$ that are their own multiplicative inverses are $\pm 1$. (Note this is only once class if $p = 2$.)

3. Recall that positive integers $a, b \in \mathbb{N}$ are called relatively prime if $\gcd(a, b) = 1$. Suppose you draw $a$ and $b$ at random from $\mathbb{N}$. What do you think the probability that they’re relatively prime is? More likely than not? Relatively rare?

   Prove that this probability is at most $3/4$. Can you get any other bounds?

4. We saw (if we got this far) that the $p$ for which $-1$ is a square mod $p$ seem to be those of the form $n^2 + m^2$.

   Investigate other squares. For which $p$ is $2$ a square mod $p$? Is this related to $p$ being expressible in a certain form? I don’t expect you to be able to prove everything. Experiment, form conjectures, prove what you can, etc. That’s the process.