**Algebra 1 : Fourth homework — due Monday, October 24**

Do the following exercises from Fulton and Harris:

5.6, 5.8, 5.11

Also do the following exercises:

**1.** Recall that for $n \geq 1$, and any field $k$, we let $\mathbf{P}^{n-1}(k)$ denote the set of lines in $k^n$.

(a) Show that the natural action of $\mathrm{GL}_n(k)$ on $k^n$ induces a transitive action of $\mathrm{GL}_n(k)$ on $\mathbf{P}^{n-1}(k)$, and compute the stabilizer of the line $k \times 0 \times \cdots \times 0$ under this action.

(b) Taking $k$ to be a finite field $\mathbf{F}_q$, use the result of part (a) to inductively compute the order of $\mathrm{GL}_n(\mathbf{F}_q)$.

**2.** (This question gives the details of one of the discussions in Monday's class.) Let $E/F$ be a finite Galois extension of fields, with Galois group $G$. Regard $E \otimes_F E$ as an $E$-algebra via the map $E \to E \otimes_F E$ given by $e \mapsto e \otimes 1$, and for each $g \in G$, define a homomorphism of $E$-algebras $\phi_g : E \otimes_F E \to E$ via $\phi_g : e_1 \otimes e_2 \to e_1 g(e_2)$.

Verify that each $\phi_g$ is a well-defined homomorphism of $E$-algebras, and that the product of the $\phi_g$ (as $g$ ranges over all elements of $G$) induces an isomorphism of $E$-algebras

$$E \otimes_F E \cong \prod_{g \in G} E.$$

**3.** Describe all the conjugacy classes in (a) $\mathrm{GL}_2(\mathbf{R})$; (b) $\mathrm{GL}_2(\mathbf{Q})$.