Lecture 1

1) Examples of vector bundles
    a. On R^1 – trivial bundle, any others?
    b. On S^1 – trivial bundle, Mobius bundle
    c. On S^2 – tangent bundle, normal bundle
    d. On RP^n and CP^n – tautological bundle
    e. Grassmannians G(2,4) – tautological bundle
2) Definition of vector bundle
    a. Locally trivializeable with linear transition functions)
3) Sections
    a. 0-section
    b. Functions as sections of trivial bundle
    c. Vector fields as sections of tangent bundle
    d. Extensions of sections
        i. Prove, using Tietze extension theorem
4) Sub-bundles, maps of vector bundles
    a. Jacobian of a smooth map
5) Constructions with vector bundles
    a. Pullback
    b. External direct sum, tensor product, hom
        i. Internal Direct sum, tensor product, hom
    c. Dual
    d. Determinant
        i. Orientations
    e. Flag bundle

Lecture 2

1) Gauss Map for embedded surfaces (or hypersurfaces)
2) Classifying map for tangent bundle of embedded manifolds
3) Classifying spaces
    a. Idea
    b. For vector bundles embedded in affine space
    c. Definition of Grassmannians, canonical bundles
    d. Proof of that Grassmannians classify
        i. Universal bundles
        ii. Construction of bundle maps (i.e. every v. bundle pulled back from universal one)
        iii. Sketch proof that bundles are isomorphic iff classifying maps are homotopic
            1. (Follows from section extension theorem)
4) Thom isomorphism (brief sketch)
    a. Suspension axiom for vector bundle on a point
    b. Suspension axiom for trivial bundle on an unpointed space
    c. Thom isom as twisted suspension axiom (valid for all vector bundles mod 2, valid for all oriented bundles with Z coeffs)