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# Chapter 20

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## CRing Project bibliography

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- [AM69] M. F. Atiyah and I. G. Macdonald. *Introduction to commutative algebra*. Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969.
- [BBD82] A. A. Beilinson, J. Bernstein, and P. Deligne. Faisceaux pervers. In *Analysis and topology on singular spaces, I (Luminy, 1981)*, volume 100 of *Astérisque*, pages 5–171. Soc. Math. France, Paris, 1982.
- [Bou98] Nicolas Bourbaki. *Commutative algebra. Chapters 1–7*. Elements of Mathematics (Berlin). Springer-Verlag, Berlin, 1998. Translated from the French, Reprint of the 1989 English translation.
- [Cam88] Oscar Campoli. A principal ideal domain that is not a euclidean domain. *American Mathematical Monthly*, 95(9):868–871, 1988.
- [CF86] J. W. S. Cassels and A. Fröhlich, editors. *Algebraic number theory*, London, 1986. Academic Press Inc. [Harcourt Brace Jovanovich Publishers]. Reprint of the 1967 original.
- [Cla11] Pete L. Clark. Factorization in euclidean domains. 2011. Available at <http://math.uga.edu/~pete/factorization2010.pdf>.
- [dJea10] Aise Johan de Jong et al. *Stacks Project*. Open source project, available at [http://www.math.columbia.edu/algebraic\\_geometry/stacks-git/](http://www.math.columbia.edu/algebraic_geometry/stacks-git/), 2010.
- [Eis95] David Eisenbud. *Commutative algebra*, volume 150 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1995. With a view toward algebraic geometry.
- [For91] Otto Forster. *Lectures on Riemann surfaces*, volume 81 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1991. Translated from the 1977 German original by Bruce Gilligan, Reprint of the 1981 English translation.
- [GD] Alexander Grothendieck and Jean Dieudonné. *Éléments de géométrie algébrique*. Publications Mathématiques de l’IHÉS.
- [Ger] Anton Geraschenko ([mathoverflow.net/users/1](http://mathoverflow.net/users/1)). Is there an example of a formally smooth morphism which is not smooth? MathOverflow. <http://mathoverflow.net/questions/200> (version: 2009-10-08).
- [Gil70] Robert Gilmer. An existence theorem for non-Noetherian rings. *The American Mathematical Monthly*, 77(6):621–623, 1970.
- [Gre97] John Greene. Principal ideal domains are almost euclidean. *The American Mathematical Monthly*, 104(2):154–156, 1997.
- [Gro57] Alexander Grothendieck. Sur quelques points d’algèbre homologique. *Tôhoku Math. J. (2)*, 9:119–221, 1957.

- [Har77] Robin Hartshorne. *Algebraic geometry*. Springer-Verlag, New York, 1977. Graduate Texts in Mathematics, No. 52.
- [Hat02] Allen Hatcher. *Algebraic topology*. Cambridge University Press, Cambridge, 2002. Available at <http://www.math.cornell.edu/~hatcher/AT/AT.pdf>.
- [Hov07] Mark Hovey. *Model Categories*. American Mathematical Society, 2007.
- [KS06] Masaki Kashiwara and Pierre Schapira. *Categories and sheaves*, volume 332 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer-Verlag, Berlin, 2006.
- [Lan94] Serge Lang. *Algebraic number theory*, volume 110 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1994.
- [Lan02] Serge Lang. *Algebra*, volume 211 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, third edition, 2002.
- [Liu02] Qing Liu. *Algebraic geometry and arithmetic curves*, volume 6 of *Oxford Graduate Texts in Mathematics*. Oxford University Press, Oxford, 2002. Translated from the French by Reinie Ern e, Oxford Science Publications.
- [LR08] T. Y. Lam and Manuel L. Reyes. A prime ideal principle in commutative algebra. *J. Algebra*, 319(7):3006–3027, 2008.
- [Mar02] David Marker. *Model theory*, volume 217 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 2002. An introduction.
- [Mat80] Hideyuki Matsumura. *Commutative algebra*, volume 56 of *Mathematics Lecture Note Series*. Benjamin/Cummings Publishing Co., Inc., Reading, Mass., second edition, 1980.
- [McC76] John McCabe. A note on Zariski’s lemma. *The American Mathematical Monthly*, 83(7):560–561, 1976.
- [Mil80] James S. Milne. * tale cohomology*, volume 33 of *Princeton Mathematical Series*. Princeton University Press, Princeton, N.J., 1980.
- [ML98] Saunders Mac Lane. *Categories for the working mathematician*, volume 5 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1998.
- [Per04] Herv e Perdry. An elementary proof of Krull’s intersection theorem. *The American Mathematical Monthly*, 111(4):356–357, 2004.
- [Qui] Daniel Quillen. Homology of commutative rings. Mimeographed notes.
- [Ray70] Michel Raynaud. *Anneaux locaux henseliens*. Lecture Notes in Mathematics, Vol. 169. Springer-Verlag, Berlin, 1970.
- [RG71] Michel Raynaud and Laurent Gruson. Crit eres de platitude et de projectivit e. Techniques de “platification” d’un module. *Invent. Math.*, 13:1–89, 1971.
- [Ser65] Jean-Pierre Serre. *Alg ebre locale. Multiplicit es*, volume 11 of *Cours au Coll ege de France, 1957–1958, r edig e par Pierre Gabriel. Seconde  dition, 1965. Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1965.
- [Ser79] Jean-Pierre Serre. *Local fields*, volume 67 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1979. Translated from the French by Marvin Jay Greenberg.

- [Ser09] Jean-Pierre Serre. How to use finite fields for problems concerning infinite fields. 2009. arXiv:0903.0517v2.
- [SGA72] *Théorie des topos et cohomologie étale des schémas. Tome 1: Théorie des topos*. Lecture Notes in Mathematics, Vol. 269. Springer-Verlag, Berlin, 1972. Séminaire de Géométrie Algébrique du Bois-Marie 1963–1964 (SGA 4), Dirigé par M. Artin, A. Grothendieck, et J. L. Verdier. Avec la collaboration de N. Bourbaki, P. Deligne et B. Saint-Donat.
- [SGA03] *Revêtements étales et groupe fondamental (SGA 1)*. Documents Mathématiques (Paris) [Mathematical Documents (Paris)], 3. Société Mathématique de France, Paris, 2003. Séminaire de géométrie algébrique du Bois Marie 1960–61. [Algebraic Geometry Seminar of Bois Marie 1960-61], Directed by A. Grothendieck, With two papers by M. Raynaud, Updated and annotated reprint of the 1971 original [Lecture Notes in Math., 224, Springer, Berlin; MR0354651 (50 #7129)].
- [Tam94] Günter Tamme. *Introduction to étale cohomology*. Universitext. Springer-Verlag, Berlin, 1994. Translated from the German by Manfred Kolster.
- [Vis08] Angelo Vistoli. Notes on Grothendieck topologies, fibered categories, and descent theory. *Published in FGA Explained*, 2008. arXiv:math/0412512v4.
- [Was97] Lawrence C. Washington. *Introduction to cyclotomic fields*, volume 83 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1997.
- [Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.